Antonine Physics AS

Topic 7 Waves

How to Use this Book

How to use these pages:

- This book intended to complement the work you do with a teacher, not to replace the teacher.
- Read the book along with your notes.
- If you get stuck, ask your teacher for help.
- The best way to succeed in Physics is to practise the questions.

There are many other resources available to help you to progress:

- Web-based resources, many of which are free.
- Your friends on your course.
- Your teacher.
- Books in the library.

This is an electronic book which you can download. You can carry it in a portable drive and access it from your school's computers (if allowed) as well as your own at home.

The Waves topic covers the properties and behaviour of waves.

We will review the wave equation, which you will have met at GCSE. We will then look at the difference between transverse and longitudinal waves. We will look at the waveforms of sinusoidal waves, including how longitudinal waves, such as sound, can be represented as graphs. We then go on to explain how polarisation happens in a transverse wave, but not a longitudinal wave. Then we discuss standing waves.

Musical instruments use standing waves, and we will look at how the musical sounds are made by varying the properties of the standing waves.

Then we investigate the wave properties of light, including diffraction, interference, and diffraction.

Topic 7 Waves

1. Wave Properties

Tutorial 7.01 Progressive Waves

All Syllabi

Contents

7.011 Wave Motion

Waves are caused by **oscillations**. Oscillations are complete to-and-fro movements, of which vibrations are one example. Another example is the oscillation of electrons, which cause radio waves. We will study oscillations in more detail with simple harmonic motion.

Energy is transferred by wave motion, but particles are not transferred.

Waves occur when a **disturbance** at the **source** of the wave causes particles to oscillate about a fixed central point. There is a maximum **displacement** from the central point, which is called the **equilibrium position**, or **average level**. When particles reach that maximum displacement, they start to move towards the central point. They pass through the central point as they move to the maximum displacement on the other side.

We can show this on a **water wave**. The particles of water oscillate up and down from the equilibrium position. The wave is travelling from left to right. **P** is going down, **Q** is at the maximum displacement, and **R** is going up (*Figure 1*).

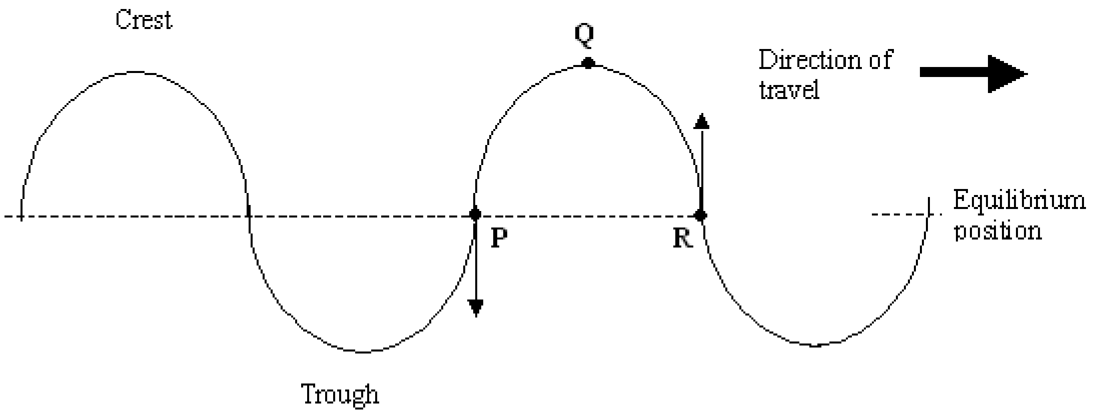


Figure 1 A transverse wave

The wave is called a **progressive** wave because it is moving in a particular direction. It is **transferring energy** from the point of disturbance, but the particles are not travelling with the wave, merely going up and down.

Ripples in solid materials like sand are NOT waves.

They are caused by the piling up of sand in the wind. They do not oscillate.

Waves can be considered to travel either as **plane wavefronts**, from a plane source or as **circular wavefronts** from a **point** source (*Figure 2*).

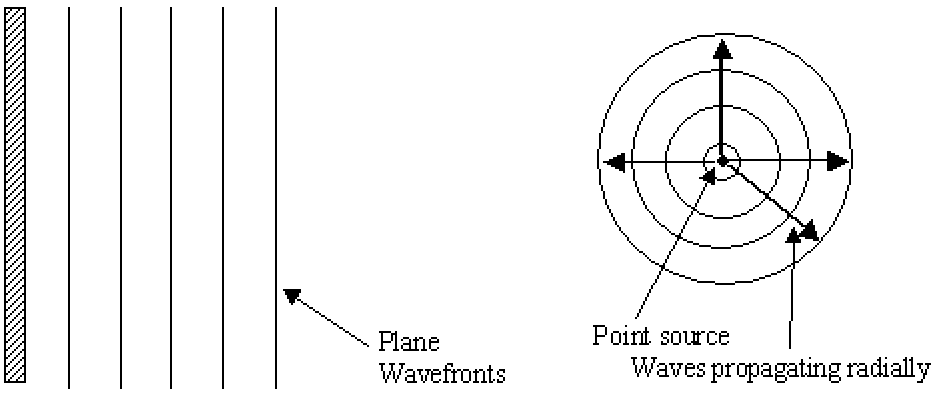


Figure 2 Wavefronts

In 3 dimensions, the waves would propagate **spherically** from a **point source**.

In plane wavefronts, all points along the wavefront are in step, or **in phase**. The direction of propagation is **perpendicular** to the wavefront.

7.012 Terms Used with Waves

Displacement of a particle is the distance at any given moment from the central or equilibrium position, i.e. the undisturbed position. It is given the Physics Code s or x, and the SI unit is metre (m). The displacement decreases the further the wave progresses from its source.

- **Intensity** of waves at a point is the **power per unit area** at that point. The energy of a wave increases as the square of its amplitude. However the energy decreases as the square of the distance from the source, which is known as the **inverse square law**. The physics code for intensity is I and the units are watts per square metre (W m^{-2}).
- **Amplitude** of a wave, code A or r, units metres (m), is the **maximum** displacement of a particle from its equilibrium position. In other words, it is the height of the wave from the average level. It is NOT the height from crest to trough. (*NB:* Be careful of the code. Here amplitude is given the code A, but in many texts, you will see a. This could be confused with acceleration.)
- **Wavelength** is defined as the distance between any two points on adjacent cycles that are **in phase**, in other words the distance between adjacent peaks or troughs. The code for wavelength is λ (lambda, a Greek letter 'l'). The units for wavelength are metre (m).
- **Frequency**, code f, has the unit hertz (Hz), and is the number of waves passing a given point every second.
- **Period** is the time taken for one complete oscillation. The code is T and the units seconds (s). Frequency is the **reciprocal** of period and is related to period by the simple equation:

$$f = \frac{1}{T}$$

.... *Equation 1*

- **Wave velocity**, code v, units metres per second (m s^{-1}), tells us the speed of **propagation** of the wave, i.e. how fast it travels. For water waves this is a few cm s^{-1}. In air, sound waves propagate at 340 m s^{-1}. For light the speed is 3×10^8 m s^{-1}. The speed of light is given the code c.

- **Mechanical** waves are produced by a disturbance in a material, or a **medium**, and can be **longitudinal** or **transverse**. Mechanical waves need a **medium** or material to travel in. In **electromagnetic** waves the disturbances are in the form of oscillating electrical and magnetic fields. They are always transverse. Electromagnetic waves can travel in a vacuum.
- The **phase** of a particle is the fraction of the cycle a particle has passed through relative to a given starting point. We describe the difference in the motion of particles in terms of the **phase difference**. This is the fraction of a wavelength by which their motions are different.
- The **path difference** between two waves is the number of cycles difference there is in the distance they have to travel.

7.013 Graphical Representation of Waves

We can show the features of waves in two ways:

- A displacement - displacement graph which shows how the wave varies in space.
- A displacement - time graph which shows how the wave varies with time.

The displacement - displacement graph shows the **physical** features of a wave (*Figure 3*).

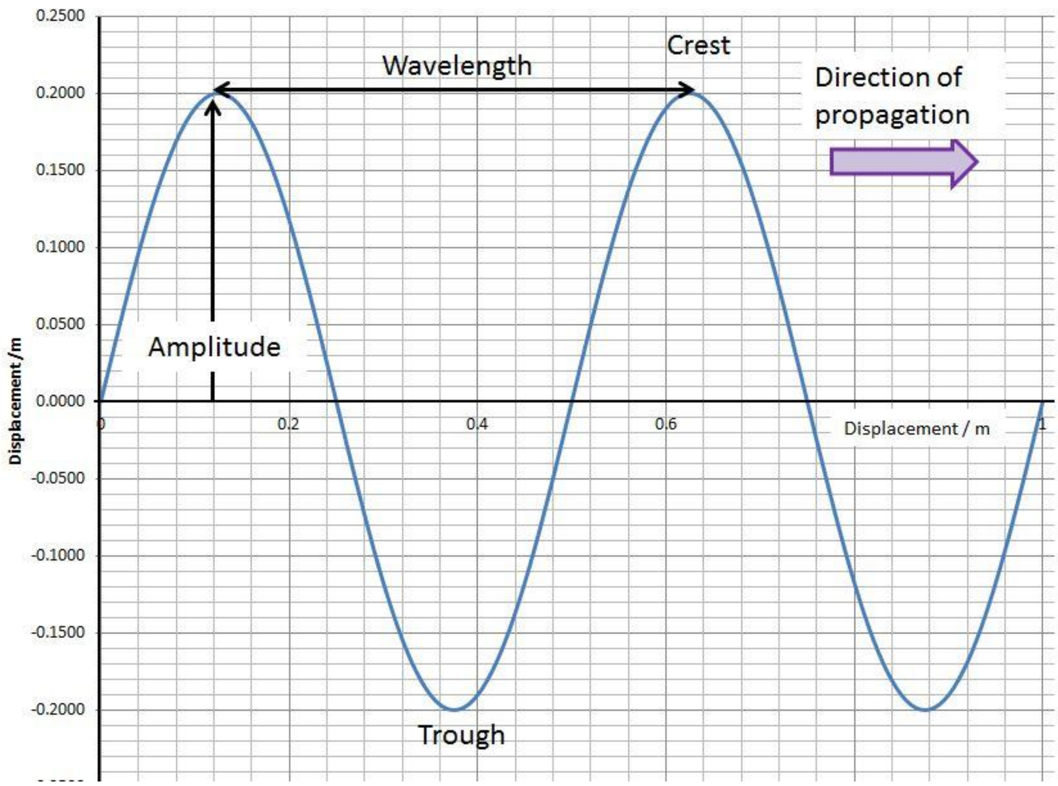


Figure 3 A displacement-displacement graph of a transverse wave

The displacement - time graph shows how the displacement varies with time, and it is like this (*Figure 4*).

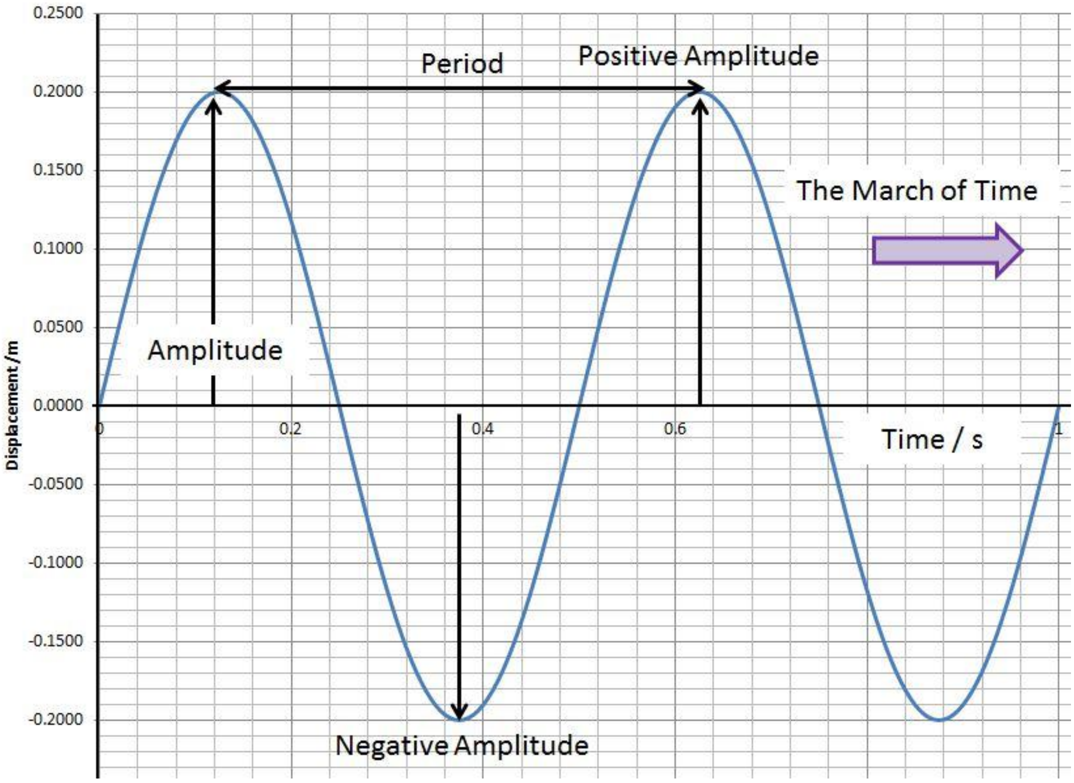


Figure 4 A displacement-time graph of a wave

You need to note these points:

- The displacement time graph does not show the physical features of the graph.
- The wavelength should not be confused with the period.
- It is not correct to call the positive amplitude a crest, not the negative amplitude a trough.

Alternating currents vary sinusoidally like the wave above.

7.014 The Wave Equation

The frequency, speed, and wavelength of any wave can be linked by the simple equation:

$$v = f\lambda \text{............ } \textit{Equation 2}$$

where v is the speed of the wave in m s^{-1}, f is the frequency in Hz, λ is the wavelength in m.

When we use this equation, we do not always have to use SI units, but it is important to be consistent.

Worked Example What is the frequency of water waves of wavelength 4 cm travelling at a speed of 1.6 m s^{-1}?
Answer Formula first: $v = f\lambda$ Rearrange: $f = v/\lambda$ f = 1.60 m s^{-1} ÷ 0.04 m = **40 Hz** (Note that the 4 cm was converted to 0.04 m to keep the units consistent. The speed (1.6 m s^{-1}) could have been changed to 160 cm s^{-1})

The wave equation is used for longitudinal and transverse waves.

Note that when we use the equation for light or radio waves, we use the code c for the speed of light - c = 3.0 × 10^{8} m s^{-1}. So, the equation is written:

$$c = f\lambda \text{............ } \textit{Equation 3}$$

7.015 Energy in a Wave

If there is a bad storm, the news reports show spectacular waves smashing against the coastline. Often there are shots of the damage that has been done to sea walls and other buildings. This is because the wind disturbs the water to make high amplitude waves.

The energy is related to the amplitude of a **mechanical** wave:

$$E \propto A^2$$

.... *Equation 4*

This can be converted into an equation:

$$E = kA^2$$

.... *Equation 5*

Where:

- E - energy (J).
- k - some constant (J m^{-2}).
- A - amplitude (m).

Therefore, when the amplitude doubles, the energy carried by the wave goes up by four times.

A more detailed treatment is studied at university level.

The physics code A can stand for **amplitude** or **area**. Be careful that you know the context in which it's being used.

The photon energy in an electromagnetic wave is given by:

$$E = hf$$

Amplitude is not involved.

7.016 Intensity

Intensity is defined as:

Energy per unit area

The physics code is I and the units are **joule per square metre** (J m^{-2}). The equation is:

$$I = \frac{E}{A}$$

.... *Equation 6*

Intensity can also be defined as:

Power per unit area

The units are **watt per square metre** (W m^{-2}). The equation is:

$$I = \frac{P}{A}$$

.... *Equation 7*

The energy has a maximum value, often written I_0, at the source. As waves **propagate**, they spread out. For each doubling of radius, the intensity goes down by 4 times. This is often called the **Inverse Square Law**. (*Figure 5*).

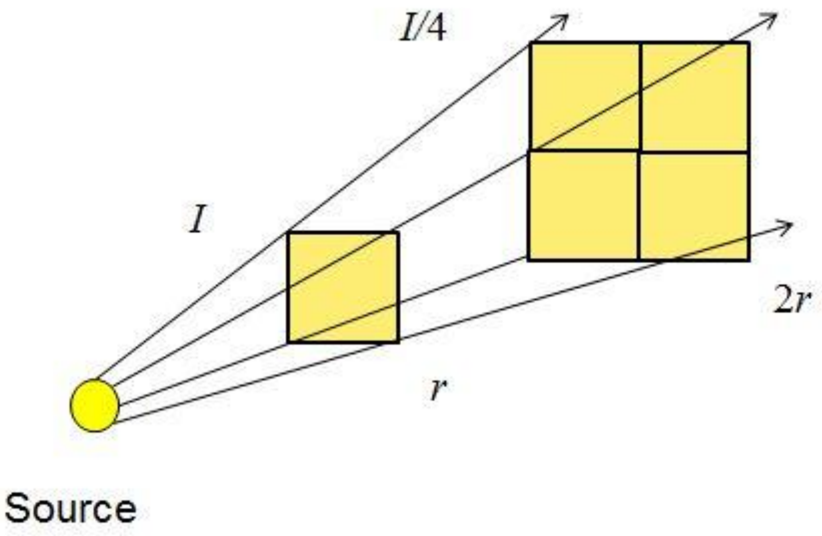


Figure 5 The Inverse Square Law

The equation for the inverse square law is:

$$\frac{I}{I_0} = \frac{1}{x^2}$$

.... *Equation 8*

This is true for all waves. Note that intensity is sometimes called **irradiance**.

7.017 Phase

When a wave is travelling, all the particles are in continuous motion. The different particles have different displacements, velocities and directions. Indeed, this is true even of adjacent particles. The **phase** of a particle is the fraction of the cycle a particle has passed through relative to a given starting point. We describe the difference in the motion of particles in terms of the **phase difference**. This is the fraction of a wavelength by which their motions are different (*Figure* 6).

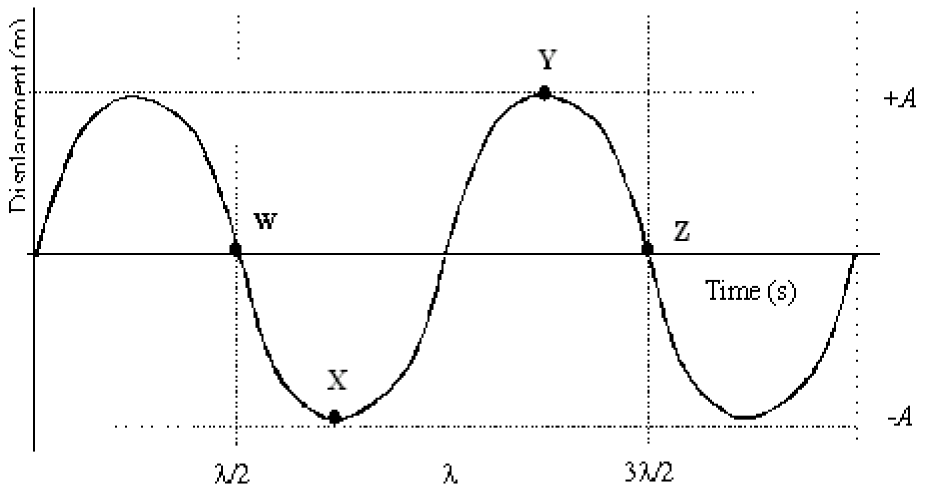


Figure 6 Phase differences on a transverse wave

Consider the two particles **X** and **Y**. **X** is at the trough of a wave, whereas **Y** is at the crest. Their directions are upwards and downwards respectively. They are half a wavelength ($\lambda/2$) **out of phase**. By linking oscillation to rotary movement, we can also describe X and Y as being 180° or π radians out of phase. We say that these particles are in **antiphase**.

W and **Z** are one wavelength, 360° or 2π radians apart. They are both at the starting point of a cycle. Their motion, including displacement, velocity and direction, is identical. We can therefore say that they are **in phase**. Particles can be any amount out of phase.

If we have two waves, we can measure their **path difference**. If the waves have a path difference of 1 wavelength (or any other whole number of wavelengths), they are in step or in phase. Waves with a path difference of 1/4 wavelength have a phase difference of 1/4 of a cycle (90° or $\pi/2$ rad). If they have a path difference of 1/2 a wavelength, the waves are in antiphase. Path difference is important when we analyse how waves **superpose**.

The phase can be calculated for any points on the wave that are d metres apart, using the relationship:

Phase angle (rad)

Horizontal distance between two points (m)

$$\phi = \frac{2\pi d}{\lambda}$$

This is "phi", a Greek letter 'f'

Wavelength (m)

.... *Equation 9*

The term d is the distance between the two points. The curious symbol ϕ is 'phi', a Greek letter 'f', used as Physics code for phase angle.

The angle in this relationship is in **radians**, where 2π rad = 360 °. Hence 1 rad ≈ 57 °.

When you use radians, you must ensure that your calculator is set to **radians**. The easiest way to lose marks is put your angle in **radians** while you are set to degrees...

It is up to you to make sure how to work your calculator.

Radians is a dimensionless unit, and some people leave it out. In these notes I will always use the shorthand **rad**.

The graph is a summary of phase relationships (*Figure 7*).

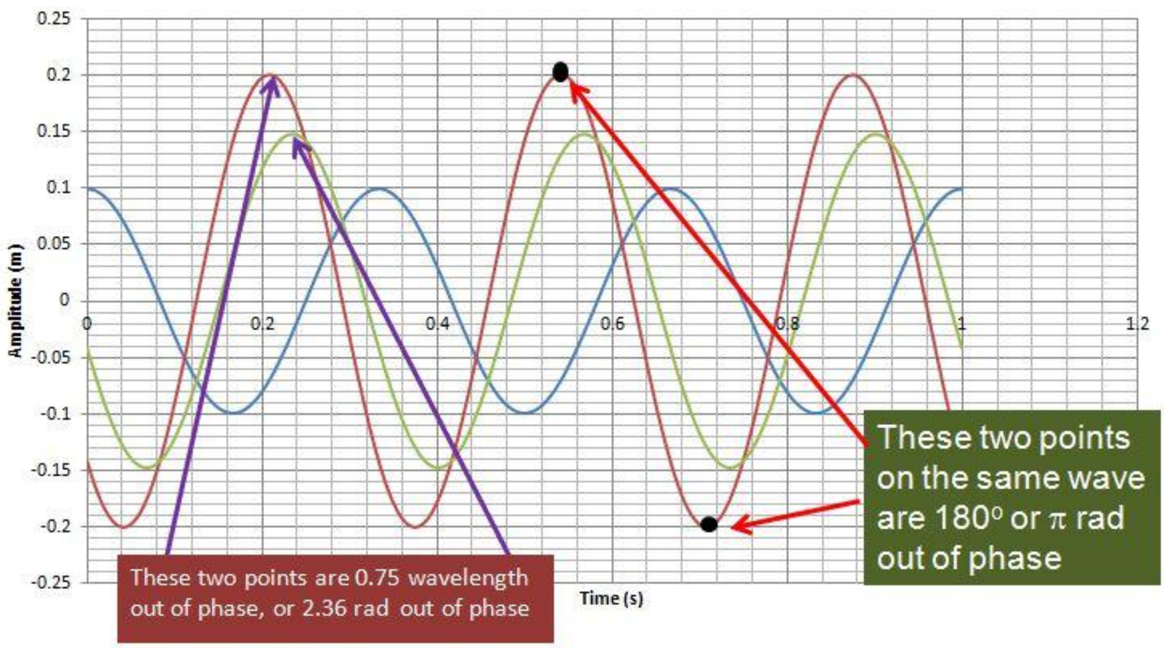


Figure 7 Phase relationships for a sinusoidal wave

7.018 Modelling Waves (Extension)

Questions that use this concept are not likely to be asked in AS exams. It is entirely possible that they will come up in A-level.

The simplest type of wave is called a **sine wave**. This is because the displacement varies with the sine of the time. The equation is shown below:

$$x = A\sin(\omega t)$$ .... *Equation 10*

The terms are:

- x - the **displacement** (m).
- A - the **amplitude** (m) which is the maximum displacement.
- ω - the **angular velocity** (rad s^{-1}).
- t - the **time** (s).

Note that x is often used for the displacement in waves. The code s can be used as well. The symbol ω is omega, a Greek lower-case letter long 'o' (ō). It represents the frequency of the wave, and is linked to the frequency by the equation:

$$\omega = 2\pi f$$

.... *Equation 11*

So, we can also write:

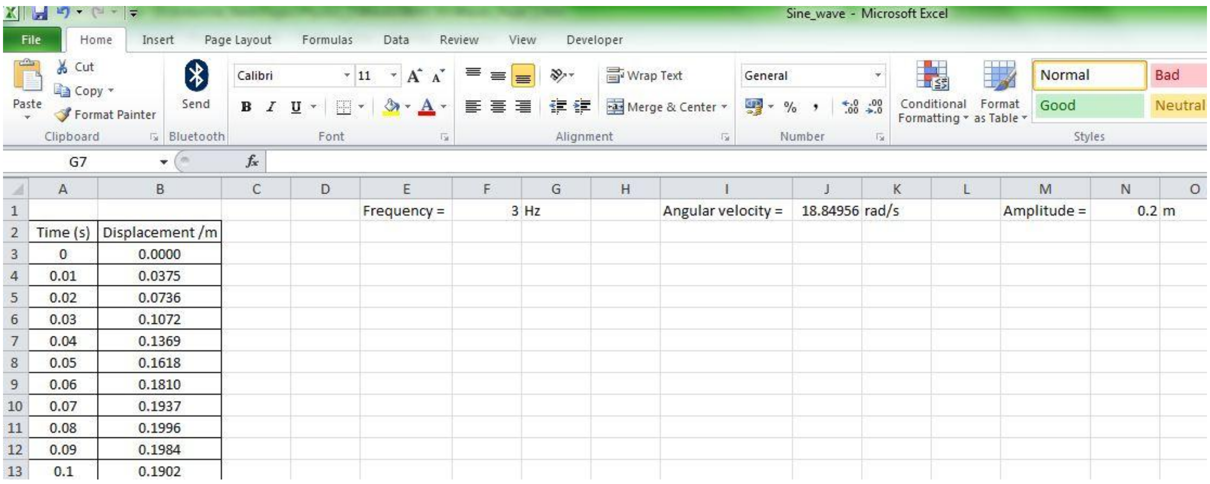
$$x = A\sin(2\pi f t)$$

.... *Equation 12*

Be careful about how you input ωt into your calculator:

$$\sin(\omega \times t) \neq \sin(\omega) \times t$$

We can use an Excel spreadsheet (other spreadsheets are available) to model the sine waves. You can try it for yourself if you are a dab hand at spreadsheets (*Figure 8*). Note that this screenshot is using an older version of Microsoft Excel ® and a more modern version will loo a bit different.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1					Frequency =	3	Hz		Angular velocity =	18.84956	rad/s		Amplitude =	0.2	m
2	Time (s)	Displacement /m													
3	0	0.0000													
4	0.01	0.0375													
5	0.02	0.0736													
6	0.03	0.1072													
7	0.04	0.1369													
8	0.05	0.1618													
9	0.06	0.1810													
10	0.07	0.1937													
11	0.08	0.1996													
12	0.09	0.1984													
13	0.1	0.1902													

Figure 8 Screenshot of Excel spreadsheet to generate data for a sinusoidal waveform

Here is a screen shot with the formulae (*Figure 9*)

	A	B	C
1			
2	Time (s)	Displacement /m	
3	0	=N$1*(SIN(J$1*A3))	
4	0.01	=N$1*(SIN(J$1*A4))	
5	0.02	=N$1*(SIN(J$1*A5))	
6	0.03	=N$1*(SIN(J$1*A6))	
7	0.04	=N$1*(SIN(J$1*A7))	
8	0.05	=N$1*(SIN(J$1*A8))	
9	0.06	=N$1*(SIN(J$1*A9))	
10	0.07	=N$1*(SIN(J$1*A10))	
11	0.08	=N$1*(SIN(J$1*A11))	
12	0.09	=N$1*(SIN(J$1*A12))	
13	0.1	=N$1*(SIN(J$1*A13))	

Figure 9 The formulae used

This spreadsheet model has a time interval of 0.01 s for a time period of 1.0 s. The sine function in Excel works with angles in **radians**. It does not work with degrees. Here is the sine wave produced with the model (*Figure 10*).

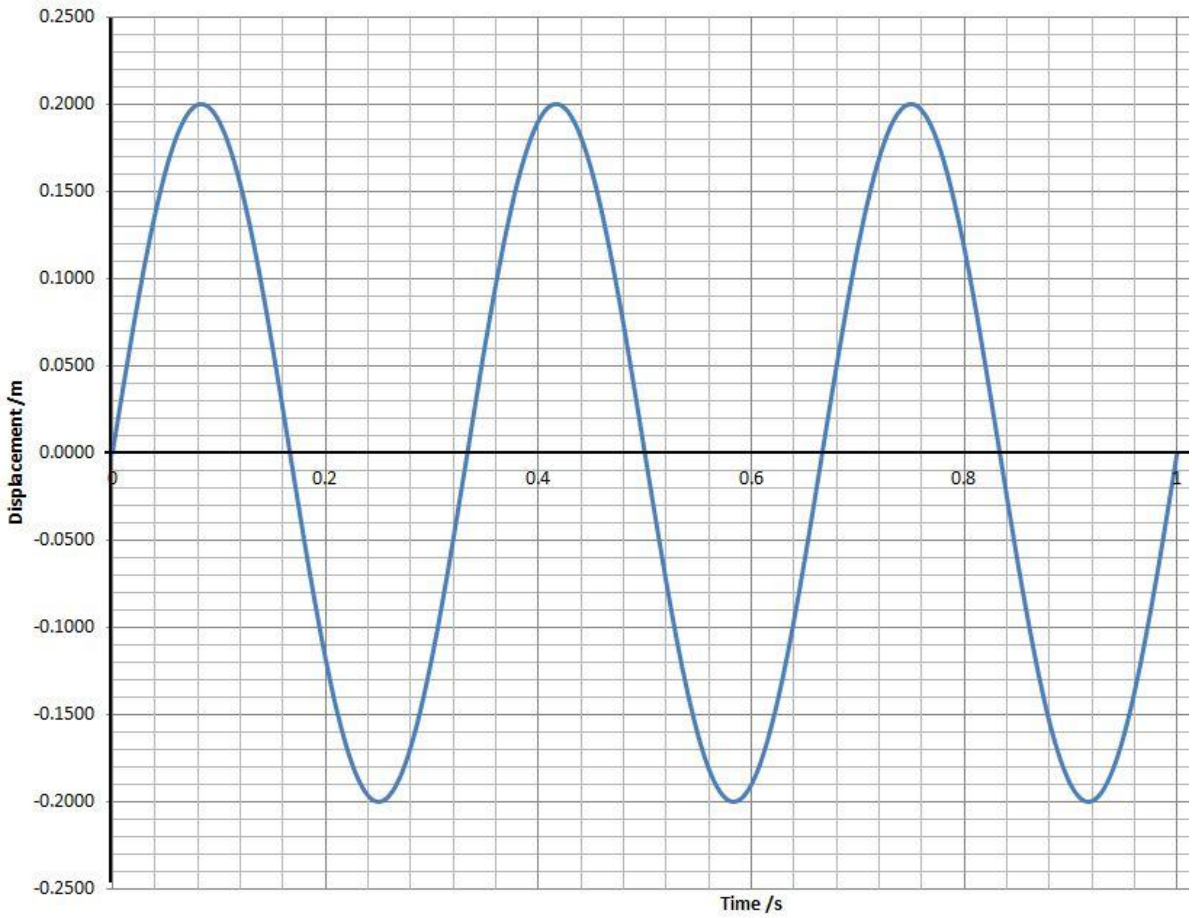


Figure 10 Sinusoidal wave produced by a spreadsheet.

Velocity of the particles that form the wave can be worked out using the gradient of the sine wave. In calculus notation, we can write:

$$v = \frac{dx}{dt} = A\omega\cos(\omega t)$$

.... *Equation 13*

Maths Window

The derivatives of trigonometrical functions are as follows:

$$\frac{d}{dx}\sin x = \cos x$$

And

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

We won't look at the tangent functions here.

If there is a constant being processed by the function, then the derivative is multiplied by the constant. In this case, we'll make the constant B. Therefore:

$$\frac{d}{dx}\sin(Bx) = B\cos(Bx)$$

And:

$$\frac{d}{dx}\cos(Bx) = -\sin(Bx)$$

For acceleration, we can write:

$$a = \frac{dv}{dt} = \frac{d_2x}{dt^2} = \frac{d}{dt}(A\omega\cos(\omega t)) = -A\omega^2\sin(\omega t)$$

......*Equation 14*

The minus sign tells us that the acceleration is towards the average level (zero displacement).

Sine waves are very closely related to:

- Circular motion.
- Simple harmonic motion.

Both of these are in the second year A level and will be discussed in Further Mechanics.

Tutorial 7.01 Questions

7.01.1

Write down three different examples of oscillations.

7.01.2

What are the features of wave motion?

7.01.3

How is energy transferred if particles do not travel with the wave?

7.01.4

What do you understand by these terms?

Displacement

Period

Frequency.

7.01.5

On a diagram of a transverse wave, mark:

Wavelength

Amplitude

Crest

Trough

Direction of disturbance.

7.01.6

A ripple tank dipper makes 8 water waves in a time of 2 s. When it is just about to make the 9th wave, the first wave has travelled 48 cm from the dipper.

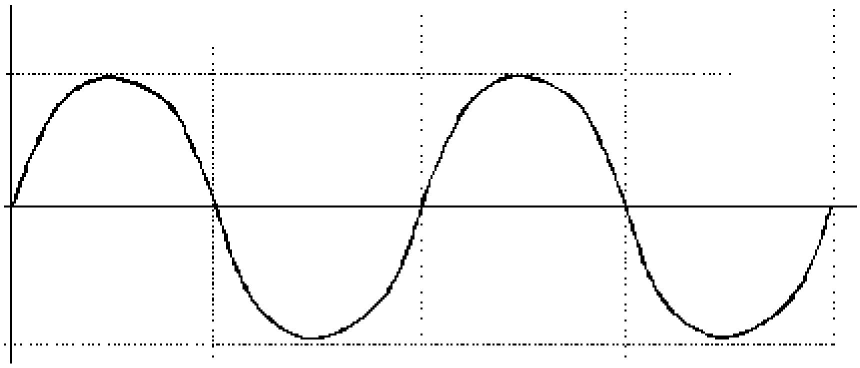
(a) What is the frequency of the waves?

(b) What is the wavelength of the waves?

(c) What is the wave speed?

7.01.7

On a diagram similar to the picture below, draw a second wave that is lagging the first wave by 90°, i.e. it's behind the first wave. Draw another wave that is π radians out of phase. Is it leading or lagging?



Tutorial 7.02 Transverse and Longitudinal Waves	
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7.023 Visible Light	7.024 Speed of Electromagnetic Waves
7.025 Polarisation of Transverse Waves	7.026 Malus' Law
7.027 Graphical Representation of Waves	7.028 Measuring the Speed of Sound
7.029 Gravitational Waves	

7.021 Progressive Wave Features

We will assume that all waves are **sinusoidal**. A sinusoidal waveform (sine wave) is the simplest kind of wave. Sound waves of sinusoidal form are rather boring to listen to. The waves made by musical instruments are more interesting, but more complex. However, it can be shown that even the most complex waveform can be broken down into sine waves. Wave motion can be analysed in terms of circular motion and simple harmonic motion (SHM). Since these topics are in the A2 Syllabus, we will not attempt to do that in these tutorials. (Originally the topic on wave properties was in the A2 syllabus, so waves could be discussed in terms of SHM.)

A **transverse** wave is one in which the **displacement** of the particles is at **90°** to the **direction of travel**. In a water wave, the particles move up and down while the wave travels horizontally. All **electromagnetic waves** are transverse.

We can show the features of a transverse wave in the diagram (not very good) below (*Figure 11*):

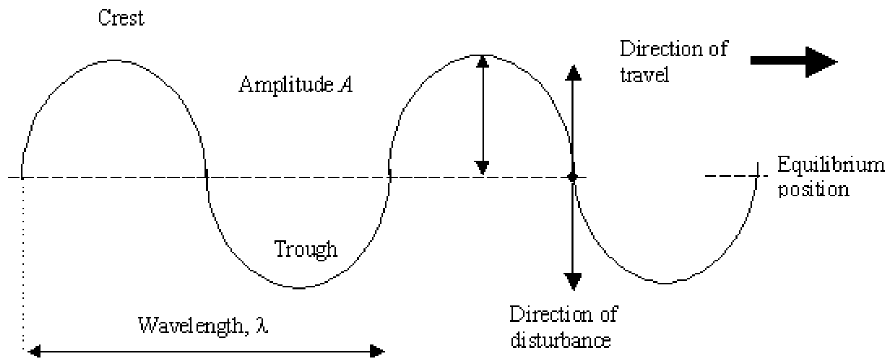


Figure 11 Diagram of a transverse wave

The **equilibrium** position in the diagram is the position that the material would take if the wave motion stopped. We could also call it the **rest** position. Both terms are used in SHM.

Strictly speaking, water waves are not transverse. They are a type of wave called a **roller** and are only transverse when the amplitude is much less than the wavelength. When the amplitude is large, the wave is no longer transverse; it takes on the characteristic shown below (*Figure 12*).

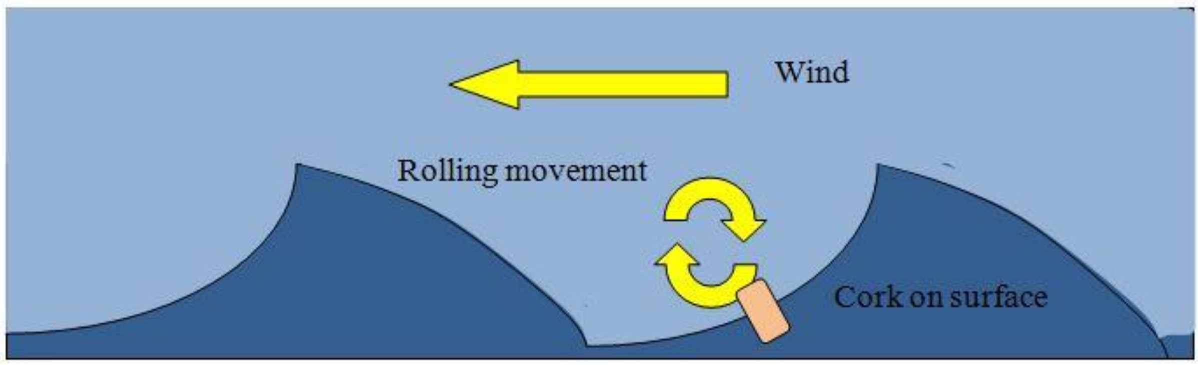


Figure 12 Water waves are rollers, not transverse.

A cork on the surface does not go up and down; it takes on a circular motion.

As the amplitude gets bigger compared with the wavelength, the crest then breaks. This often happens near the shore because the bottom part of the wave is travelling more slowly than the top. Surfers use the rolling nature of water waves.

In **longitudinal** waves, the displacement is **parallel** to the direction of travel of the wave. There are regions of high pressure, **compression**, and regions of low pressure, **rarefaction**. In a sound wave the air molecules move forwards and backwards; where they are squashed together, a compression results, where they are forced further apart, there is a rarefaction. Like all mechanical waves, a **medium** or material is required. The speed of sound in air is 336 m s^{-1}, in water 1400 m s^{-1}, in steel it is 6000 m s^{-1}. Other examples of longitudinal waves include some kinds of earthquake waves (the **pressure** or P-wave). We can see the features of a longitudinal wave in the diagram below (*Figure 13*)

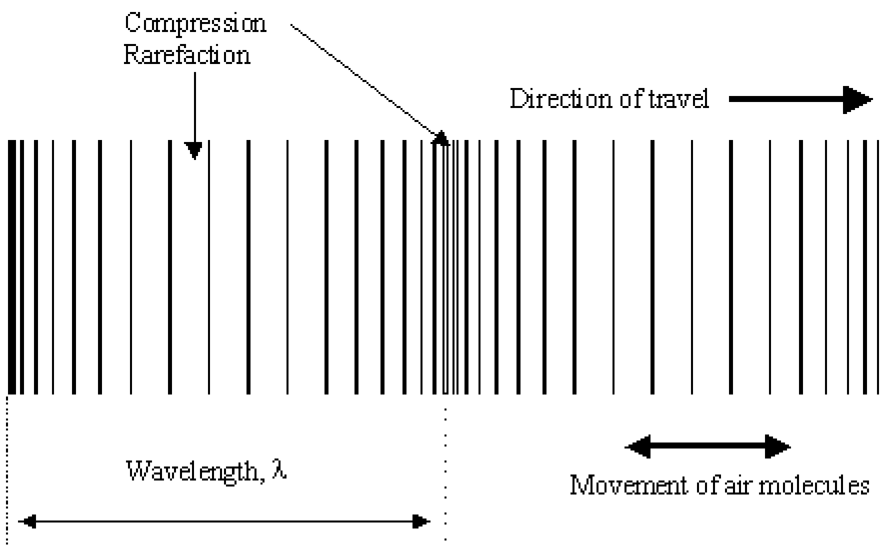


Figure 13 A longitudinal wave

7.022 Electromagnetic Waves

Electromagnetic waves have the following properties:

- They are transverse.
- They consist of an electrical wave component and a magnetic component.
- They travel in straight lines at 3.0×10^8 m s^{-1}.
- They can travel in a vacuum.
- The magnetic and electrical waves are at 90° to each other and are in phase.

The diagram (*Figure 14*) shows the idea:

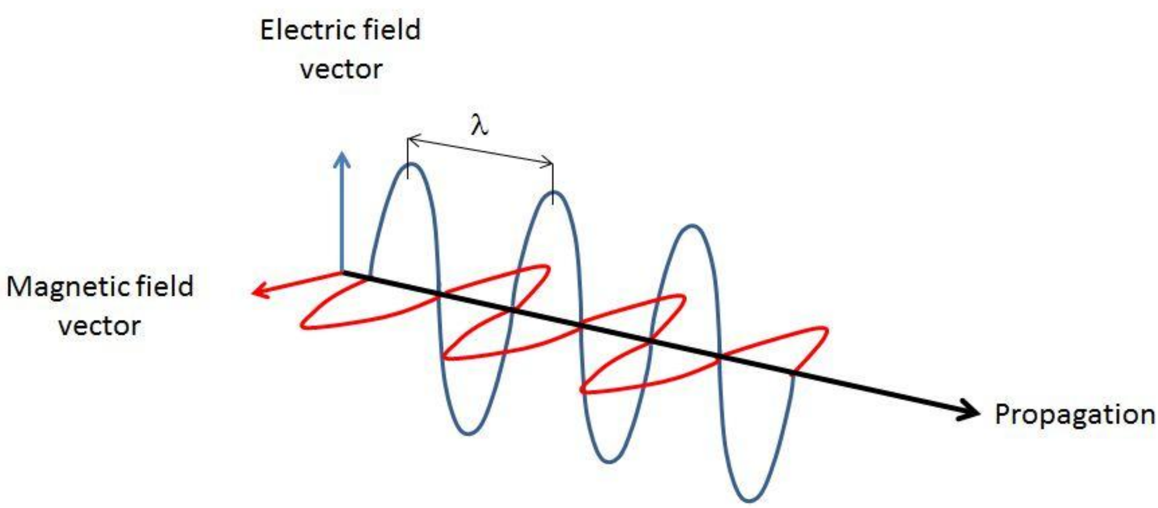


Figure 14 An electromagnetic wave.

Electromagnetic waves form a large family of waves. The EM spectrum is shown in the diagram (*Figure 15*):

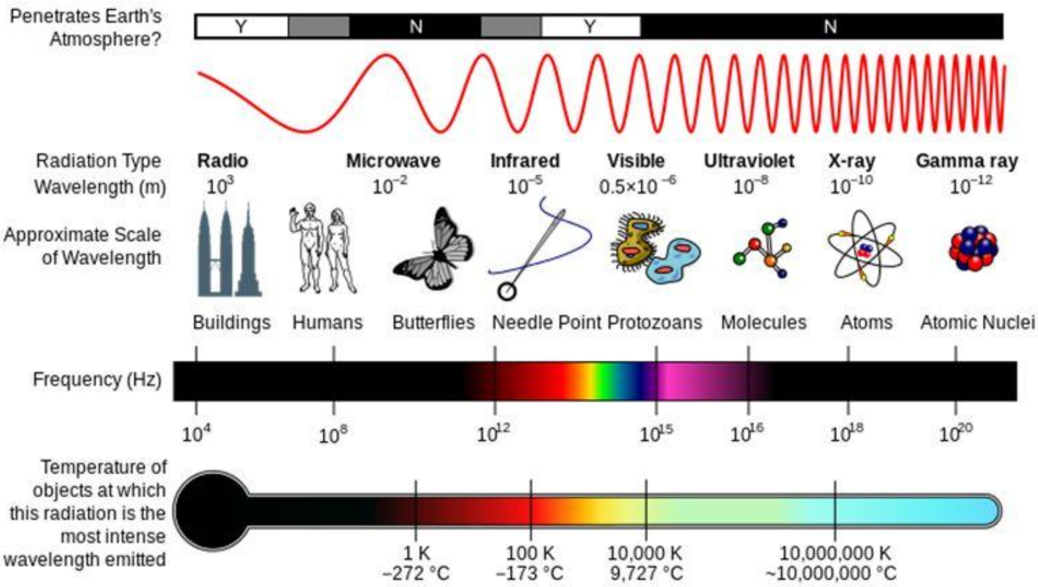


Figure 15 The electromagnetic spectrum (Image courtesy of Inductiveload, NASA. Wikipedia)

The energy in the wave is inversely proportional to the wavelength:

$$E \propto \frac{1}{f}$$

.... *Equation 15*

This becomes:

$$E = \frac{hc}{\lambda}$$

.... *Equation 16*

We have seen this in the context of photon energy:

- E - energy (J)
- h - Planck's Constant = 6.63×10^{-34} J s.
- c - speed of light = 3.0×10^{8} m s^{-1}.
- λ - wavelength (m).

In this topic, we are counting EM radiation as a wave, although we know that they travel in trains of waves called **photons**.

This equation applies to electromagnetic waves, NOT to mechanical waves.

7.023 Visible Light

Visible light is a very small part of the electromagnetic spectrum. We can see wavelength from about 300 nm to 600 nm. Below 300 nm we have ultra-violet. Many animals, e.g. insects and fishes can see in ultra-violet. We cannot see UV. No animals can see infra-red, which has a wavelength longer than 700 nm, and this allows biologists to observe animals at night without disturbing them.

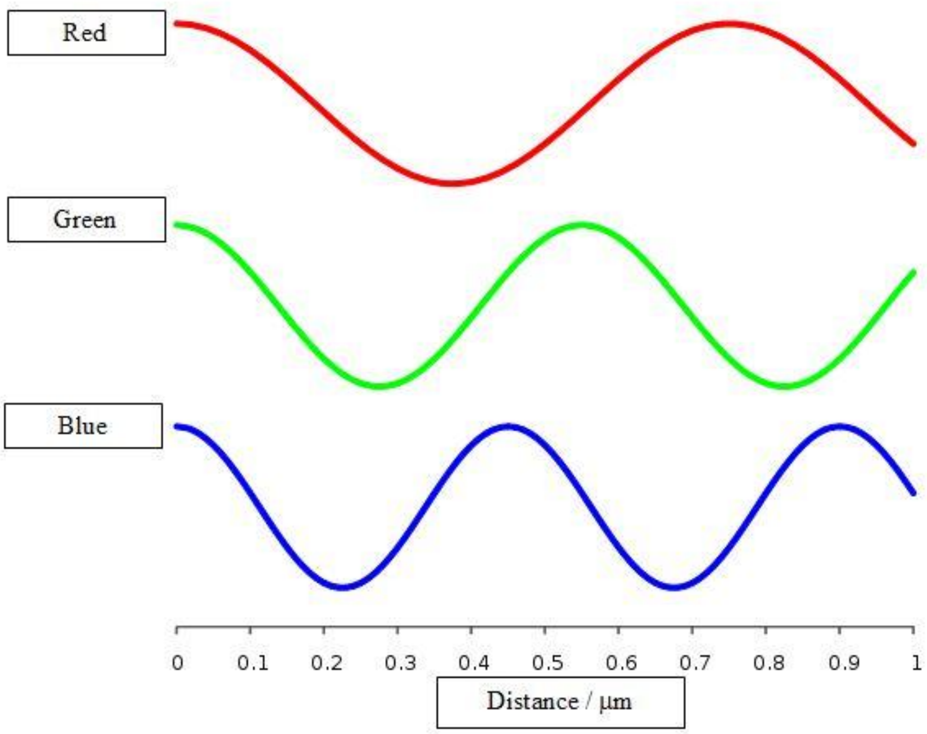


Figure 16 Different wavelengths of visible light

The picture (*Figure 16*) shows relative wavelengths of the primary colours: red, blue, and green.

If the light rays are super-imposed on each other, we get:

- Red + blue = magenta.
- Red + green = yellow.
- Green + blue = cyan.
- Red + green + blue = white.

This process is called **colour addition** (*Figure 17*).

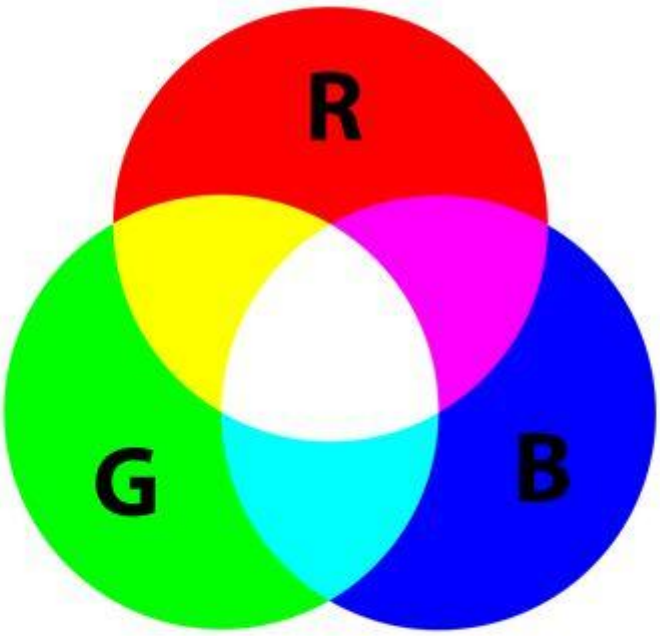


Figure 17 Colour addition Image by courtesy of SharkD. Wikipedia)

The artist's primary colours are red, blue, and yellow. This is because mixing of artists' colours works by colour **subtraction**.

7.024 Speed of Electromagnetic Waves

(Extension, Scottish Advanced Higher Syllabus)

The speed of light is given to a large number of significant figures as:

$$2.99792458 \times 10^{8} \text{ m s}^{-1}$$

It is a **fundamental constant** on which other constants rely.

We use $c = 3.00 \times 10^{8}$ m s^{-1} to 3 significant figures. But how is it worked out? Initially it was carried out by astronomical observations by Olaus Roemer, a Danish astronomer, in 1676. He observed the eclipses of the moons of Jupiter. In the 19th Century, several physicists used rotating mirrors to determine the speed. The details of such experiments are not needed here, but you can see some video clips of how it's done. See HERE.

The Scottish theoretical physicist James Clerk Maxwell used the complex Maxwell's equations to derive this simple relationship:

$$c = \frac{1}{\sqrt{(\varepsilon_0 \mu_0)}}$$

.... *Equation 17*

The terms are:

- Speed of light - c = 3.00 × 10^{8} m s^{-1}.
- Permittivity of free space - ε_0 ("epsilon-nought") = 8.85 × 10^{-12} F m^{-1}.
- Permeability of free space - μ_0 ("mu-nought") = 4π × 10^{-7} H m^{-1} » 1.257 × 10^{-6} H m^{-1}.

(Epsilon is a Greek letter 'e'. Mu is a Greek letter 'm'.)

Worked Example

Use the equation:

$$c = \frac{1}{\sqrt{(\varepsilon_0 \mu_0)}}$$

to show that the speed of light is approximately 3.0×10^8 m s^{-1}.

Answer

$c = 1 \div (8.85 \times 10^{-12}$ F m$^{-1} \times 4\pi \times 10^{-7}$ H m$^{-1})^{0.5}$

= **2.999×10^8 m s^{-1}** (which is pretty close to 3.0×10^8 m s^{-1})

We can work out the value for ε_0 using a parallel plate capacitor. We can use a current balance to work out μ_0.

7.025 Polarisation of Transverse Waves

Polarisation is a feature of **transverse waves** only. Longitudinal waves are never polarised. We say that a wave is **plane polarised** if all the vibrations in the wave are in a **single** plane, which contains the direction of propagation of the wave. Suppose we have a rope and make waves down it. We could make waves in any direction we liked. But if we made waves through a narrow vertical slit, we would find that the waves would only pass through if they were vertical. This would be a **vertically** polarised wave.

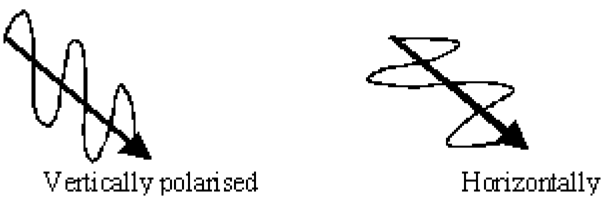


Figure 18 Vertically and hoizontally polarised transverse waves

Light waves are easily polarised using **polaroid** filters (*Figure 19*). Light waves, like all electromagnetic waves, consist of an **electric field** component perpendicular to a **magnetic field** component, which are always in phase. We normally consider only the electric field component in polarisation, because the electrical effects are those that dominate. The unpolarised waves are normally oriented in any direction.

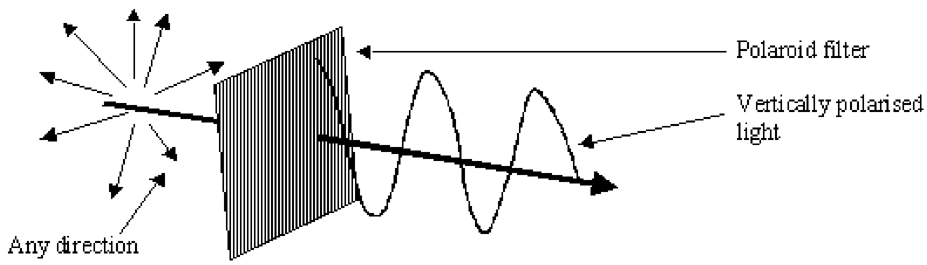


Figure 19 Polarising light vertically

If two polaroid filters are mounted such that they are parallel, the light will pass through both the first at which point it is vertically polarised, and then through the second (*Figure 20*).

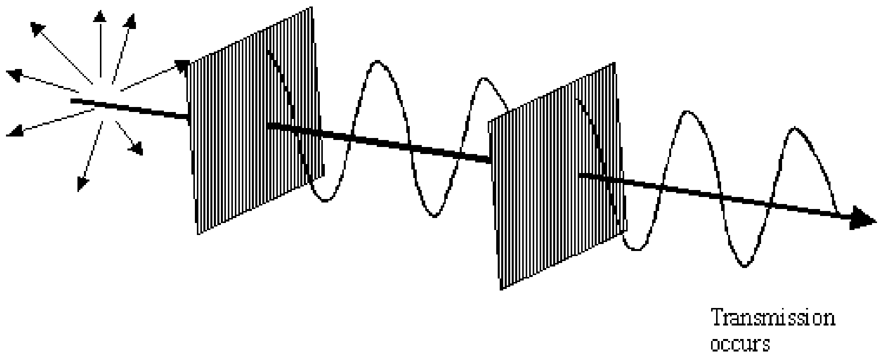


Figure 20 Light passing through two parallel polarising filters.

If the two filters are crossed, so that the **transmission planes** are at 90° to each other, the vertically polarised light gets blocked, because it cannot pass the horizontal transmission plane. No light passes (*Figure 21*).

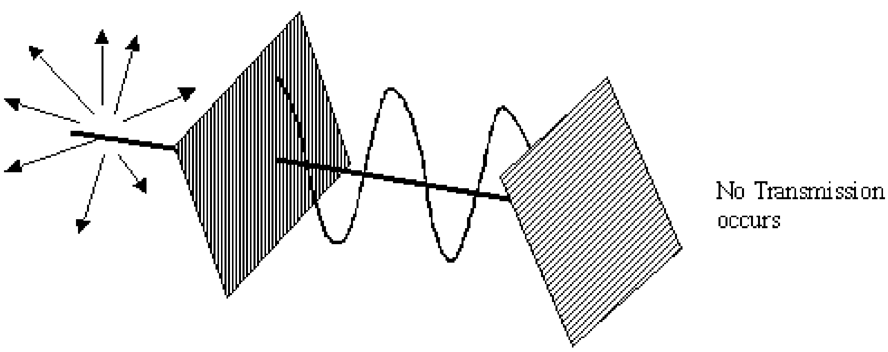


Figure 21 Light does not pass through crossed polaroids

Crossed polaroids are found in liquid crystal displays on calculators and petrol pumps.

7.026 Malus' Law (Extension)

The quantitative nature of polarisation is summed up by Malus' Law.

The intensity (power per square metre) of light passing through crossed polaroids is related to the angle of rotation by the following relationship:

$$I = I_0 \cos^2\theta$$

.... *Equation 18*

The experiment is carried out by measuring the angle between the two polaroids. The intensity is represented by the output voltage of a solar cell. The experiment uses the electric field vector of the EM wave. The measured intensity is the result of the transmitted electric field. Some of the electric field vector is absorbed. The idea is shown in the diagram below (*Figure 22*).

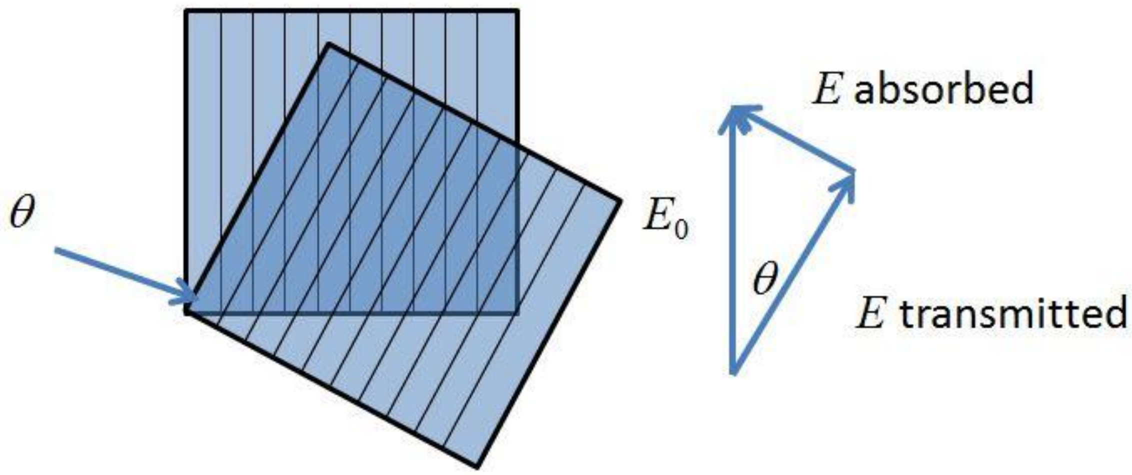


Figure 22 Measuring the angle between Polaroids

The apparatus is set up like this (*Figure 23*):

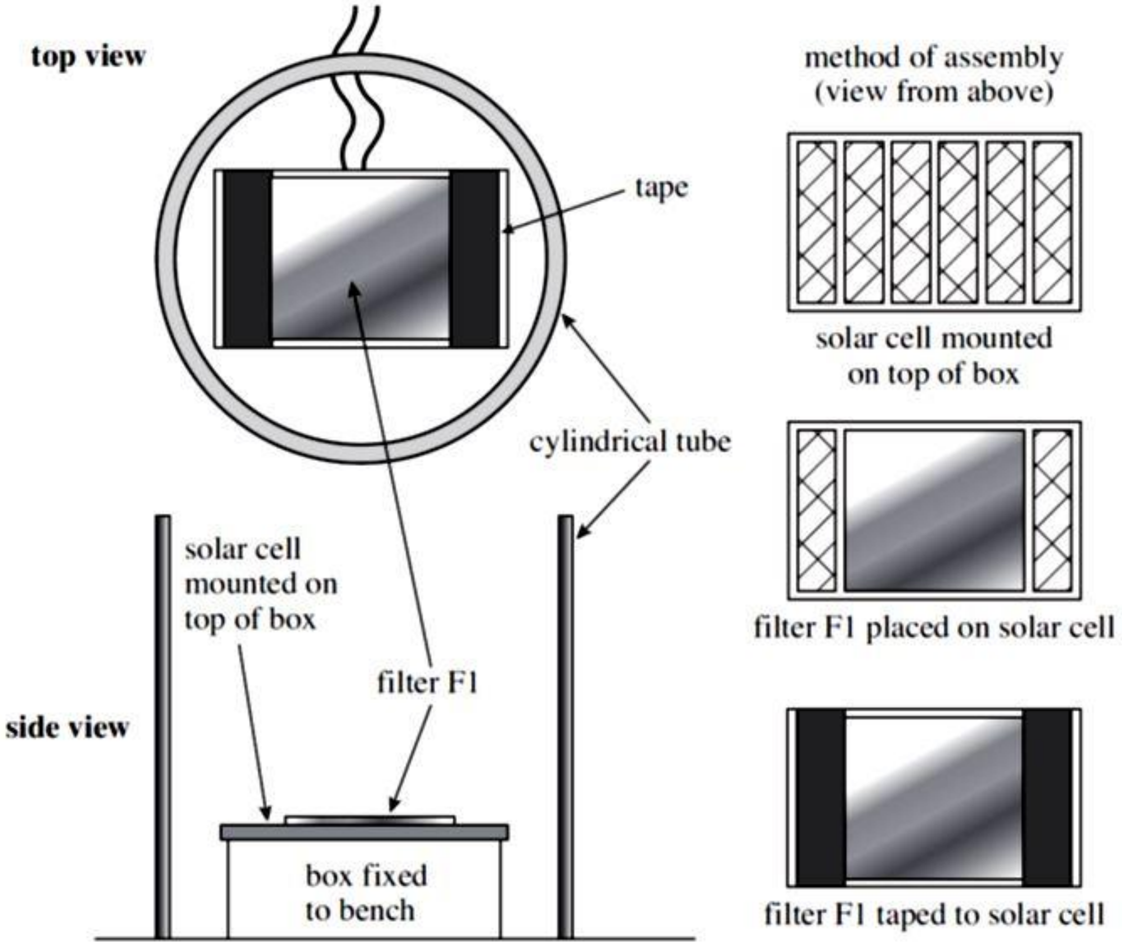


Figure 23 Experiment to investigate Malus' Law (Source: AQA Specimen practical paper)

In this experiment the intensity of the light is represented in the output of the solar cell. As you change the angle of the second polaroid, the output of the solar cell changes. You take readings every 20 degrees.

Data modelling shows what the graph should look like, if Malus' Law is followed (*Figure 24*)

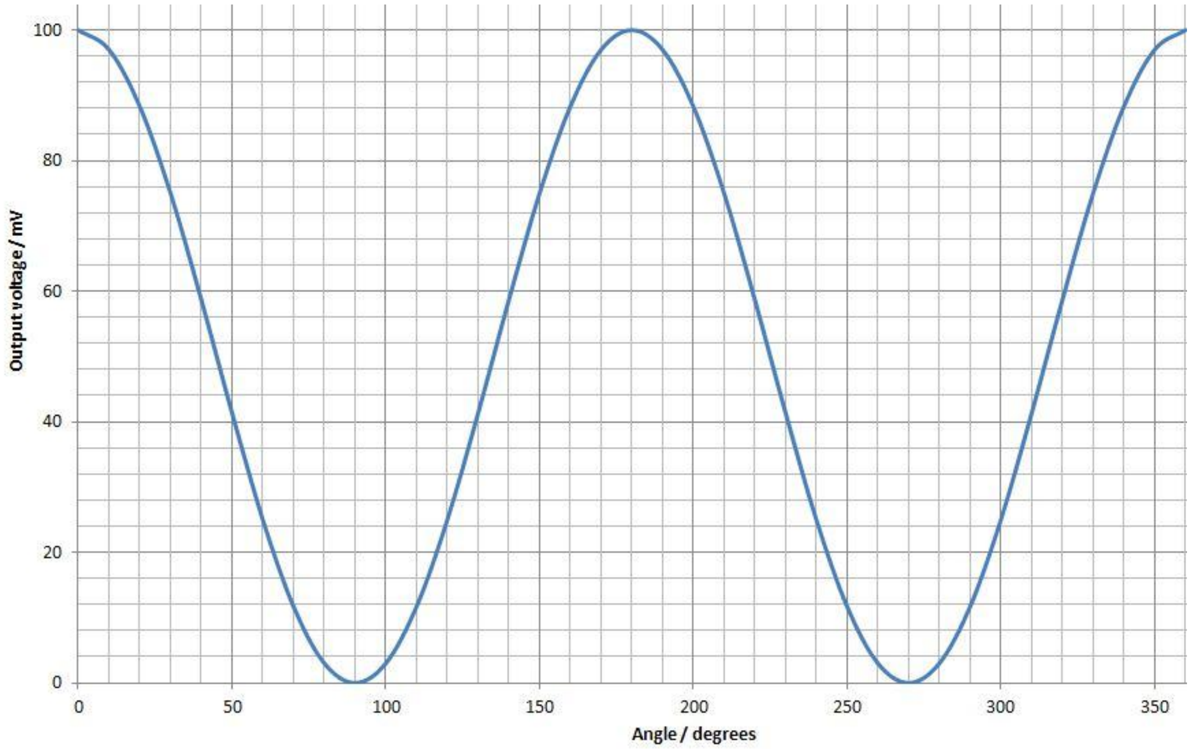


Figure 24 Theoretical Data from Malus' Law plotted as a graph.

It is quite challenging to get decent data from this experiment. At least two repeat readings should be made and an average taken. Also, the model above assumes that the crossed polaroid filters block out all the light. In reality they do not.

A much more detailed treatment of polarisation is given in the A-level Topic 15 *Supplementary Material*.

7.027 Graphical Representation of Waves

We can show that both longitudinal and transverse waves can be represented by a displacement-distance graph. If we take a snapshot of a wave at any instant, we see (*Figure 25*):

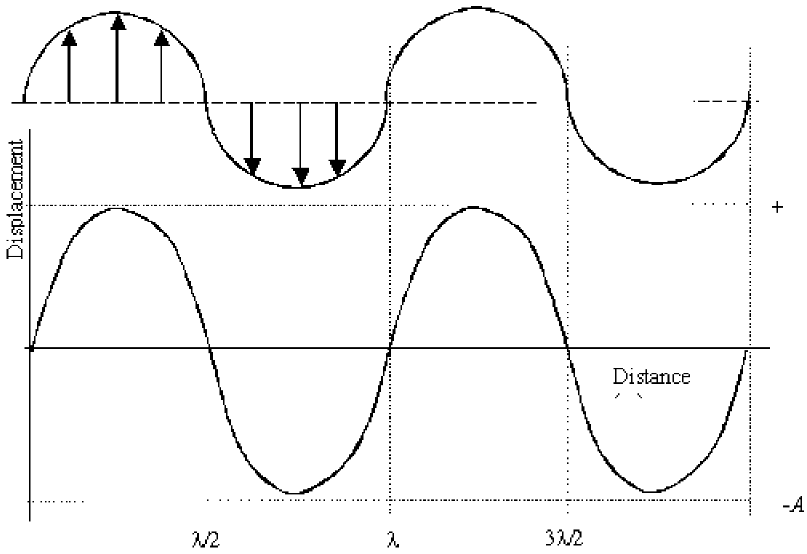


Figure 25 Transverse wave

For a **transverse** wave we see that the graph looks very similar to the actual wave.

For a **longitudinal** wave the graph is not so easy to see.

Let us look at the air molecules in their *undisturbed* positions and compare them as a sound wave passes by (*Figure 26*).

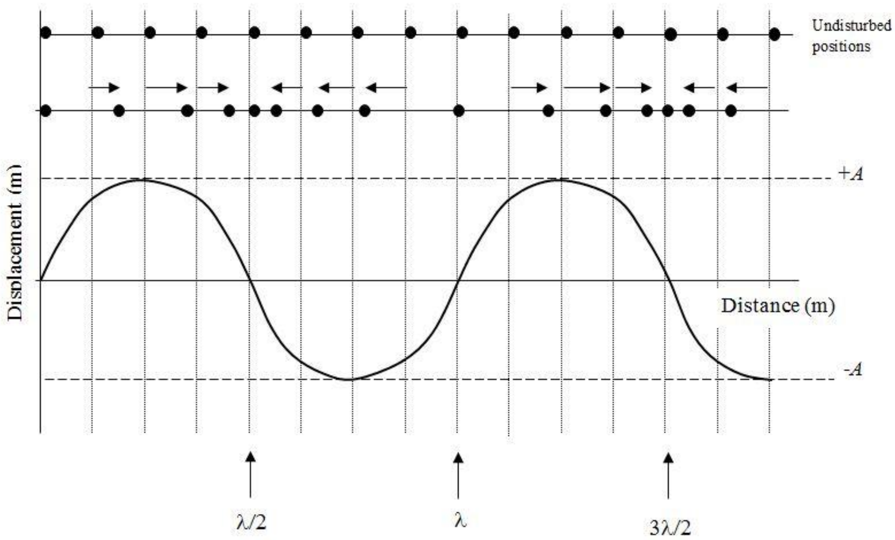


Figure 26 Displacement of air molecules in a sound wave

If we plot **displacement** on the y-axis and **distance** on the x-axis, we get the same graph to what we had before. The shape is a **sine wave.**

If we plot a displacement-time graph for a single particle we see (*Figure 27*).

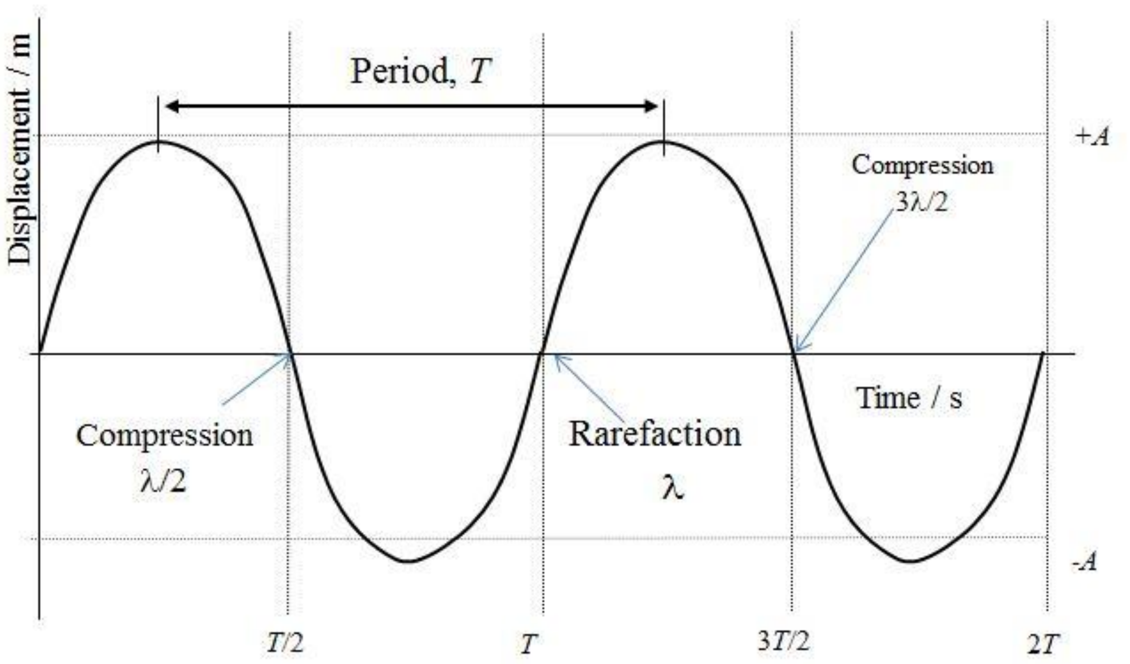


Figure 27 Displacement-time graph for a single particle

This is true whether we have a longitudinal or transverse wave. If we connect a microphone to a CRO, the CRO displays a **displacement-time trace**. It is important that we do not confuse the **displacement-distance** with the **displacement-time** graph. The latter tells us nothing of the wavelength, only the period (hence the frequency) of the wave.

The simplest shape of graph we see is the **sine wave**. The sine wave equation links **wave motion** with **simple harmonic motion**. Sine waves in sound are very boring to listen to. The **quality** of the sound gives us important clues as to the source. Sounds made by different musical instruments have very different wave patterns, even if the notes sounded and the volumes are the same.

7.028 Measuring the Speed of Sound

There are several ways of measuring the speed of sound. You may have seen a simple method of clapping two boards together and listening for the echoes. (I hated that one because I couldn't get the rhythm right!) Another one was to set off a loud firework at one end of the school playing field and time from the flash to the bang. The results were crude to say the least.

At A-level you will do a method that involves measuring the speed of sound using a CRO. You will learn more about the CRO in a later tutorial. Your tutor will demonstrate it.

This method involves the use of a signal generator driving a loudspeaker. Microphone 1 picks up the sound waves at a fixed point. Microphone 2 can be moved. This is shown in the diagram below (*Figure 28*).

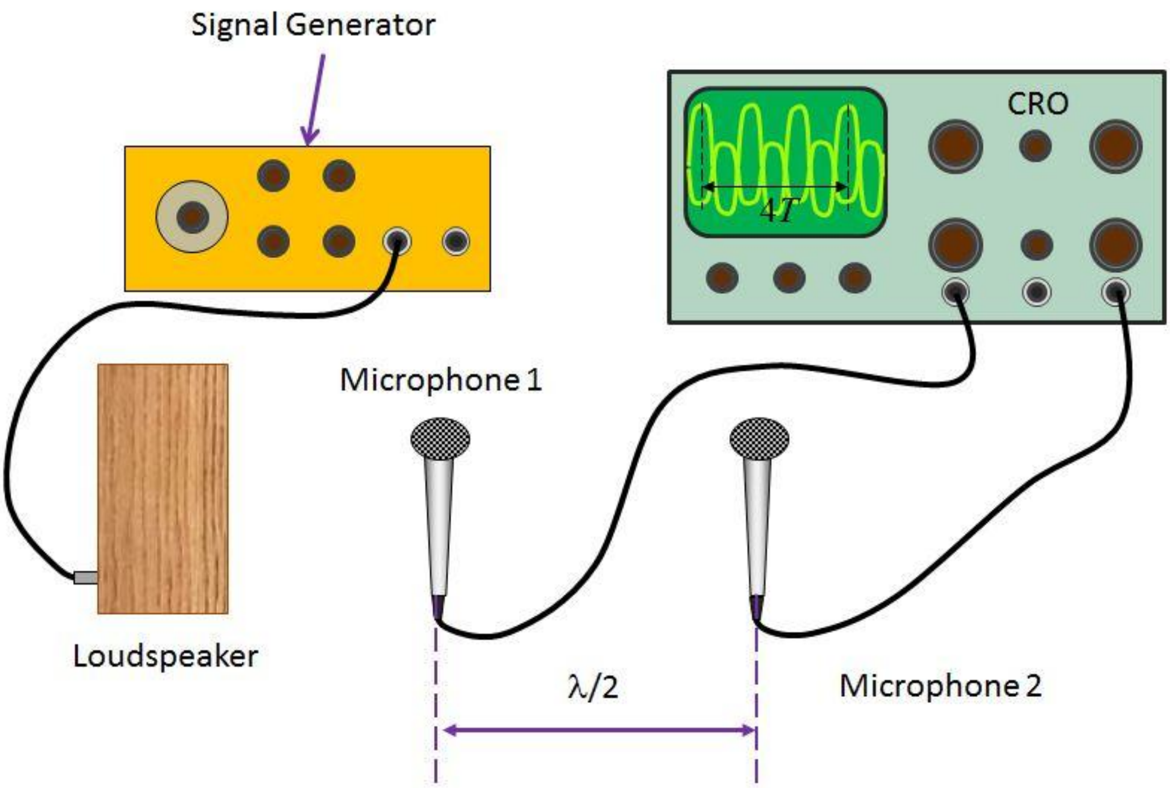


Figure 28 Measuring the speed of sound

You move Microphone 2 until the waves are 180° out of phase. You will have to alter the voltage gain for the channel that Microphone 2 is connected to. This is because the intensity falls as you move the microphone (double the distance, the intensity goes down to a quarter). You then measure the half-wavelength. In this diagram, there are four cycles, so if you measure four time periods, you will have a lower uncertainty.

The greatest uncertainty comes from the scale of the signal generator itself. They are rarely accurately calibrated. You can measure the true frequency using either the frequency measured on the CRO, or, if your department has one, a frequency meter.

You need to take at least five different frequencies, preferably more, and measure the half-wavelength. Of course, you double the half-wavelength to give you the full wavelength.

If we plot wavelength against frequency, we get a graph like this (*Figure 29*).

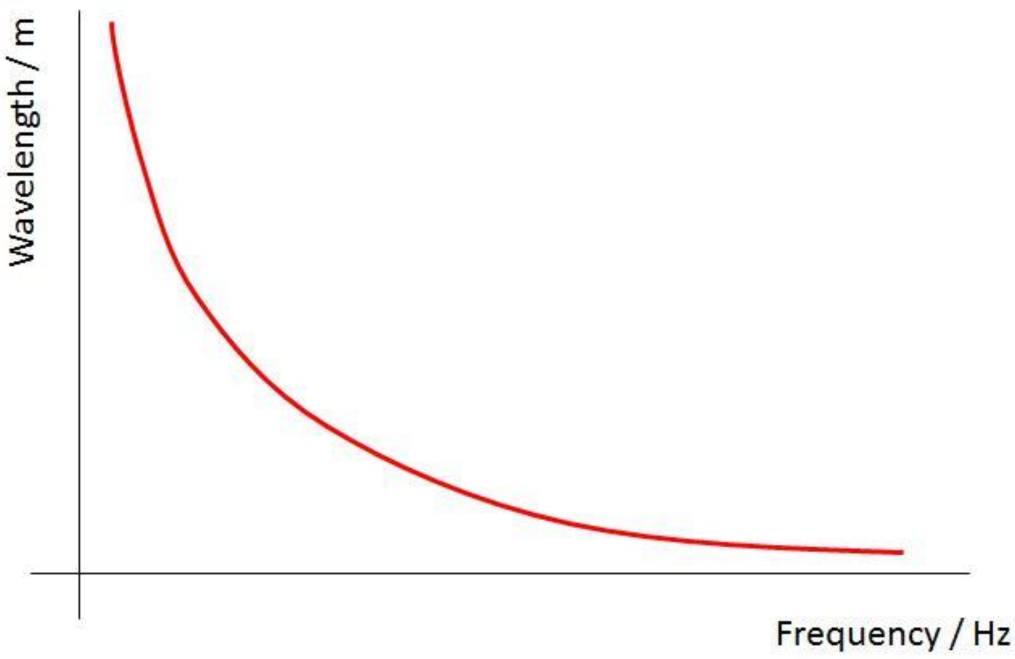


Figure 29 Graph of wavelength against frequency.

This makes sense, as $c = f\lambda$. Therefore $\lambda = c/f$. If we plot wavelength against $1/f$, we will get a straight line.

Since $T = 1/f$, we can get a straight line from a plot of wavelength against time period (*Figure 30*).

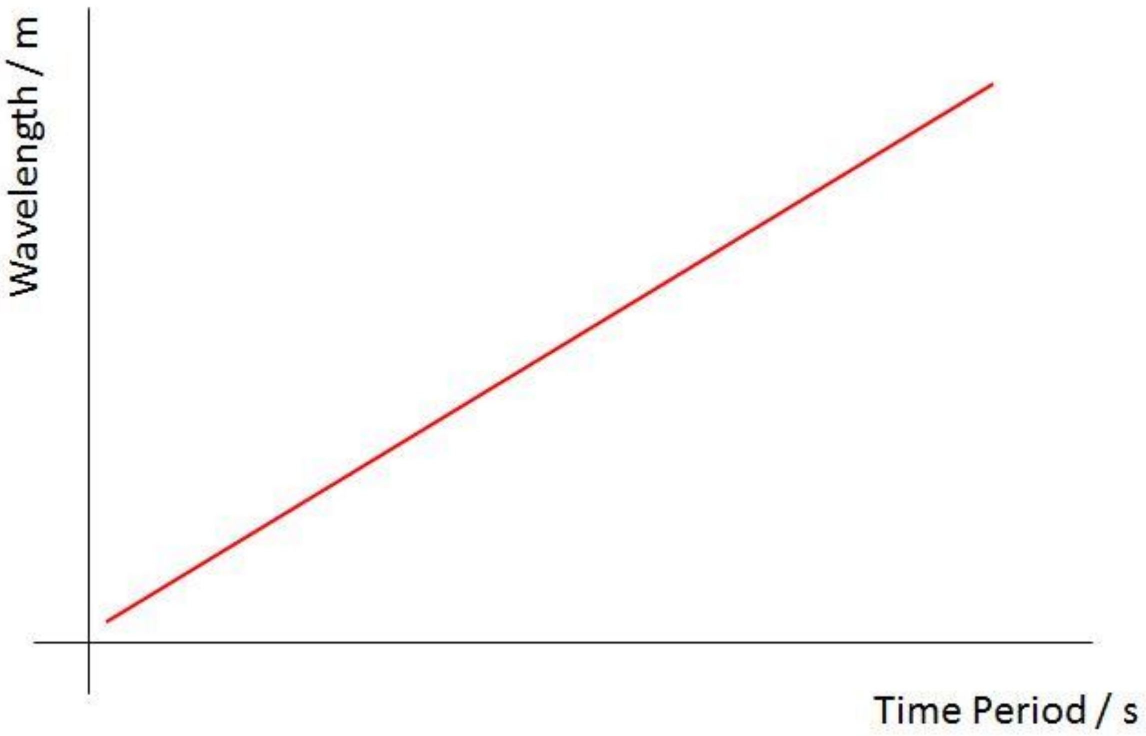


Figure 30 Graph of wavelength against time period gives the wave speed

This is a simple distance time graph, so the gradient will give you the speed of sound.

This is a **Core Practical** for the Edexcel Syllabus.

7.029 Gravitational Waves (Extension)

The notion of **gravitational waves** has been around since 1893 when first proposed by Oliver Heaviside (1850 - 1925). They were also proposed in 1905 by Henri Poincaré (1854 - 1912). The theory was developed by, and is credited to, Albert Enstein (1879 - 1955) in 1916, using his Theory of General Relativity. They can only be observed when a disturbance arises from the interaction of two or more very heavy objects, like stars. A single star on its own will not set off a gravitational wave.

The general features of gravitational waves are:

- Speed is 3.0×10^8 m s^{-1}.
- They are transverse.
- They follow the normal wave equation ($c = f\lambda$).
- The period of such wave varies from a few milliseconds to millions of years.

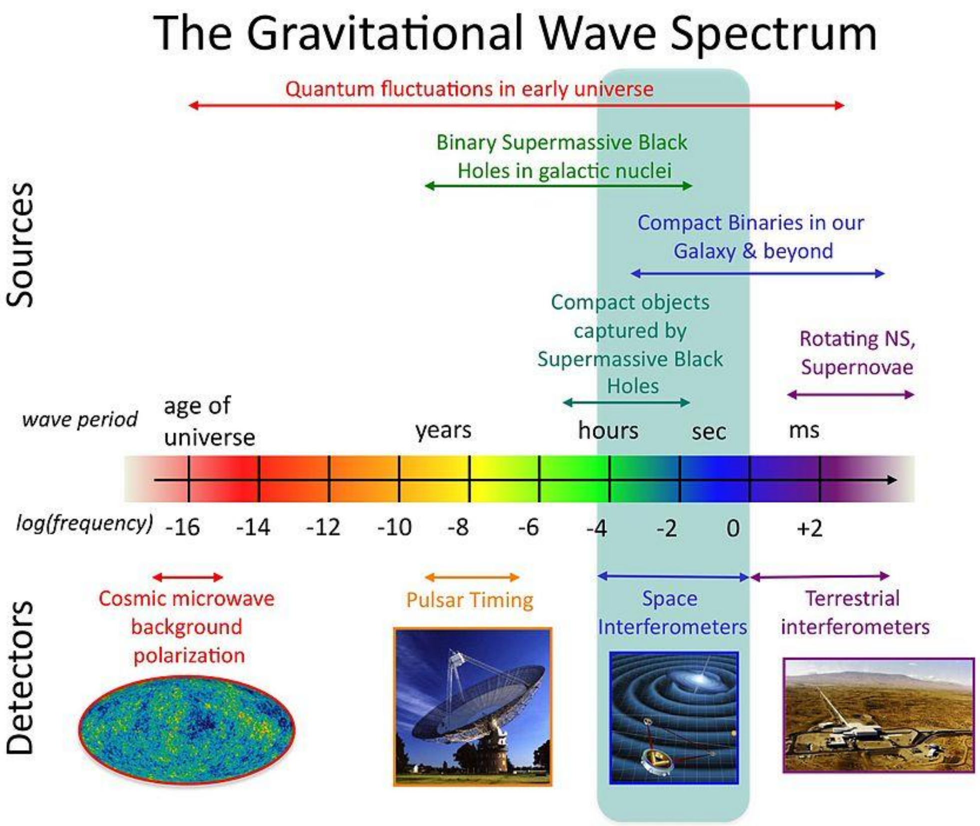


Figure 31 Gravitational wave spectrum (Image Credit: NASA Goddard Space Flight Center)

Gravitational waves from two merging black holes were detected in 2015 and announced in 2016. **Electromagnetic waves** are associated with **photons**; **gravitational waves** are associated with **gravitons**.

Tutorial 7.02 Questions

7.02.1

Write down two similarities and two differences between transverse and longitudinal waves. Give one example of a transverse wave and one example of a longitudinal wave.

7.02.2

For a wave of wavelength 95 nm, calculate:

(a) The frequency.

(b) The photon energy in J and eV.

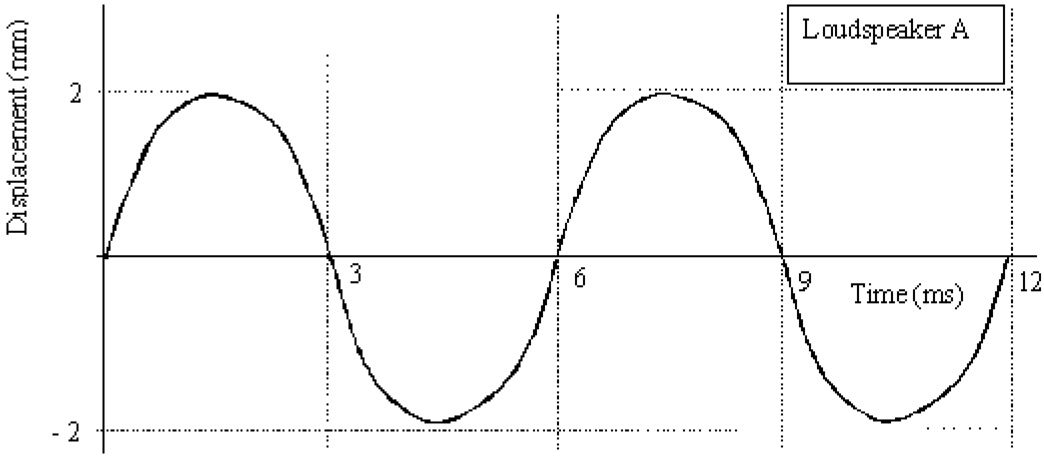
7.02.3

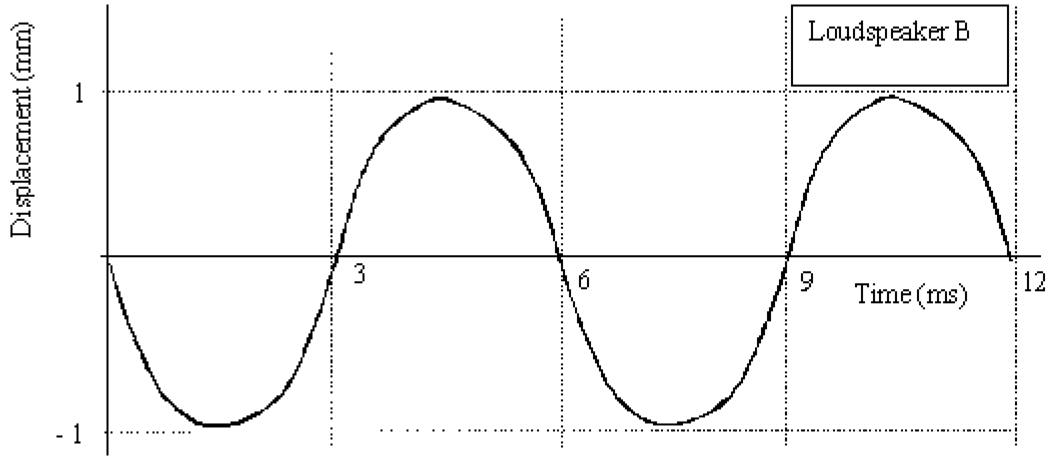
Radio aerial rods must be in the correct plane, vertical or horizontal in order to work properly, otherwise the signal is weak. Use the information in 7.025 to explain why this is the case.

7.02.4 is on the next page.

7.02.4

The diagrams show the variation with time t of the displacement x of the two identical cones of loudspeakers A and B in air.





Calculate:

(a) the frequency of the vibration of the speaker cones.

(b) the phase difference between the speaker signals.

(c) What kind of wave is being produced in the air by each speaker?

(d) Which speaker produces the loudest sound? Explain your answer.

(e) The speed of sound in air is 340 m s^{-1}. What is the wavelength of the sound waves?

2. Wave Behaviour

Tutorial 7.03 Superposition of Waves

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7.031 Wave Interaction

We shall now look at what happens when two waves interact. If two jets of water interact, they will mix and there are collisions between droplets causing a change in speed and direction. This does not happen with waves. If two waves interact, a new wave is temporarily formed, after which the two waves carry on with exactly the same properties as before, as if nothing had happened. The waves are **superposed**.

Superposition can only be applied to waves of the **same kind**. Light and sound waves cannot superpose; light and X-rays can. Let us look at two waves of different wavelengths crossing and superposing (*Figure 32*).

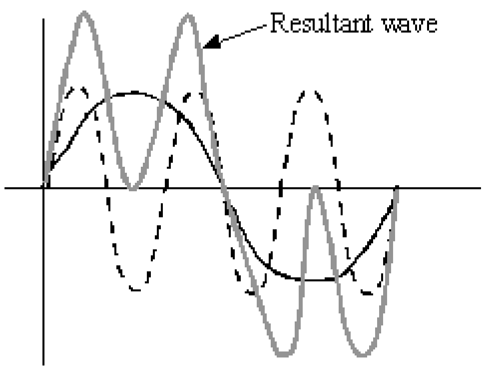


Figure 32 Waves superposing

The **resultant** wave can be worked out by the **vector sum** of the two waves. The principle of superposition of waves can be used to explain the presence of beats in sound, interference effects and standing waves.

We can use the superposition of waves to explain **interference**. When two waves meet, the amplitude of the resultant wave will not only depend on the amplitude of the two waves, but also their **phase relationship**. Let us look at two waves of equal amplitude superposing (*Figure 33*).

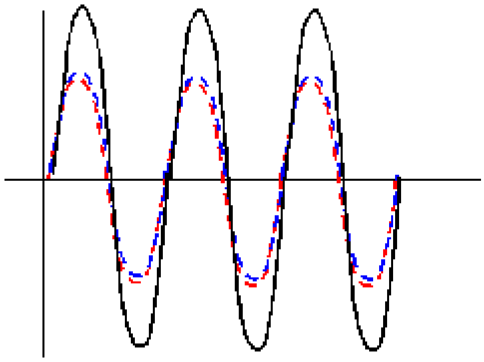


Figure 33 Two waves in equal amplitude and wavelength superposing

In this case the waves are **in phase**. The resultant wave is double the amplitude of the original waves. This is called **constructive** interference or **reinforcement**. If the waves are 180° (*p* radians) out of phase, the waves cancel each other out (*Figure 34*).

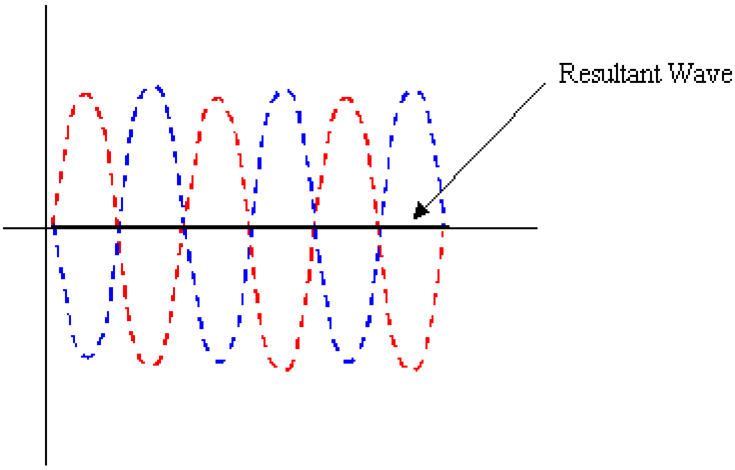


Figure 34 Cancellation

This is called **destructive** interference or **cancellation**. If the phases are different to these values, the resultant amplitude are between these two extremes.

The diagram below (*Figure 35*) shows the resultant of two waves that are 3/4 cycle (270 ° or 1.5 π rad) out of phase.

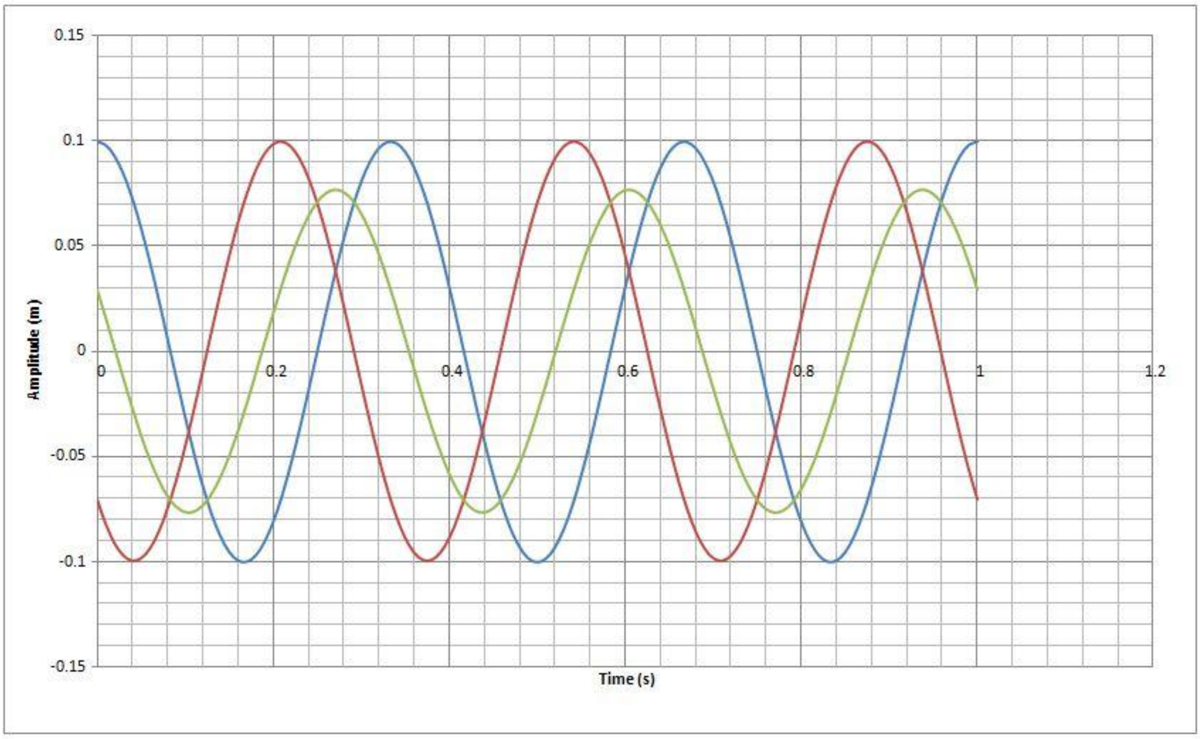


Figure 35 Wave 1.5 π rad out of phase

The red and blue waves are superposing. The green wave is the resultant.

Superposition of waves is the principle that explains:

- Interference.
- Diffraction patterns.
- Standing waves.
- Musical instruments.
- Acoustics in buildings.

7.032 Sound Pictures

Sound waves reflect. We hear the reflections as echoes. Bats use echoes to locate their prey, by building up a "sound picture" from the patterns produced by reflected sound pulses. You can see the enlarged ears that pick up the reflected sound (*Figure 36*).

Figure 36 Enlarged ears on a bat (Picture by Vehlo, Wikimedia Commons.)

The waves produce the sound picture by **superposing**, as the reflected waves interact with each other. The pattern of superposed waves will be altered by the presence of a moth (or similar juicy food item), and the animal will use that to aim at its target. (In some places, bats have given up with this, and have reverse-evolved to their original form, a wingless, mouse-like animal that eats seeds.)

Stereophonic music reproduction works in a similar way (*Figure 37*).

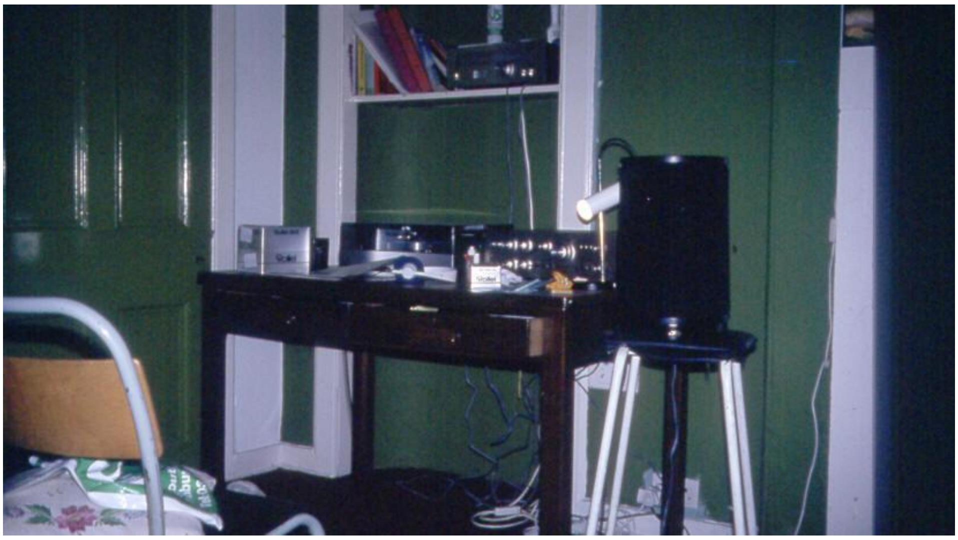


Figure 37 A Hi-Fi Stereo system (rather dated)

If your stereo system is set up properly, i.e. the speakers are in the same plane and you sit at the apex of a triangle, you will be able locate the position of the instruments being played. The same effect is heard when you wear headphones (*Figure 38*).

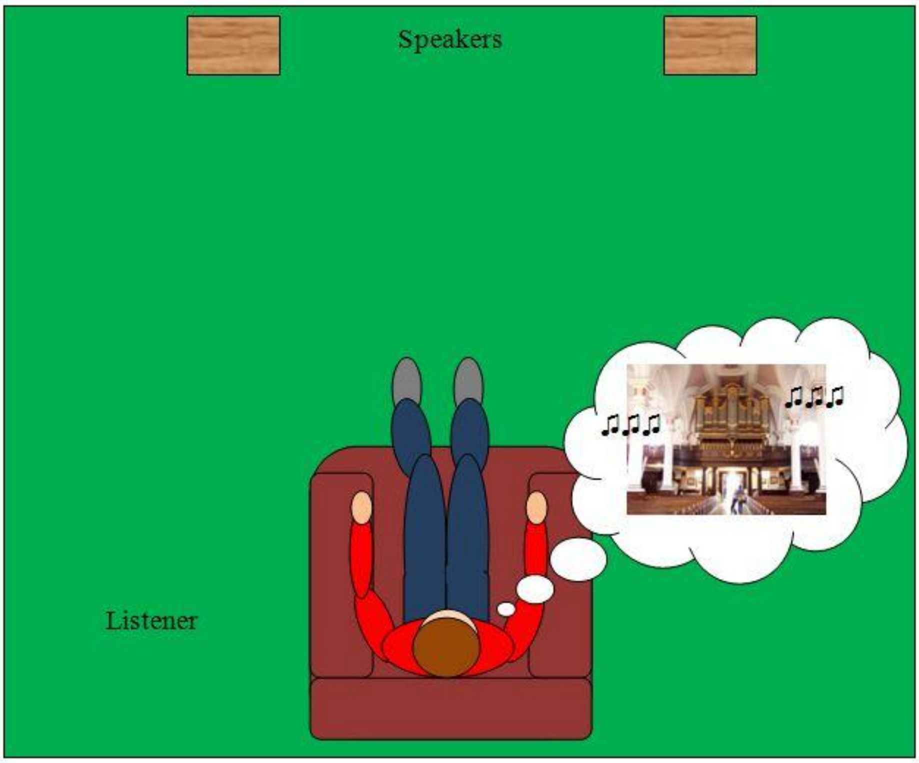


Figure 38 Listening to a stereo music recording

The apparent placing of the instruments arises from the pattern of **superposed** waves. As well as the quality and beauty of the music, a stable stereo image makes the pleasure of listening even greater. The effect is somewhat diminished by placing one speaker on a bookshelf, and one behind the settee.

Reflected sound waves and the way they superpose affects the **acoustics** of buildings. In some halls and large rooms, the acoustics can be bad. There may be spots where you cannot hear the musicians or actors properly. There may be areas where the sound is fainter that it should be, while in other areas it is louder. In some cathedrals, a sound may be accompanied by echoes that last three or four seconds; music or speech becomes inaudible or unpleasant to listen to. In mediaeval times, when these churches were built, nobody knew a thing about acoustics. Nowadays it is possible to alter acoustics so that certain echoes are absorbed by sound deadening material. This can result in substantial improvements.

If the room has too much sound deadening material, the sound can be rather "dull" or "flat". Scientific studies of audio equipment are carried out in **anechoic chambers,** but nobody would use one as a listening room. An anechoic chamber is shown below (*Figure 39*).

Figure 39 An anechoic chamber (Picture by Tlotoxl, Wikimedia Commons)

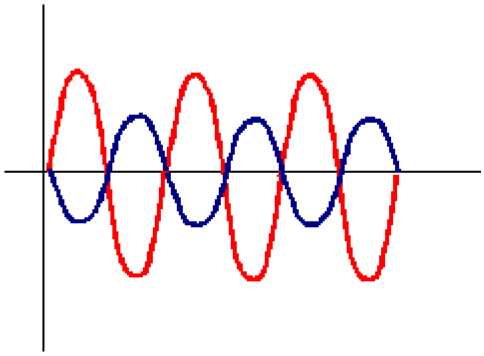
Tutorial 7.03 Questions

7.03.1

Light and water waves are both waves. Will they superpose? Explain your answer.

7.03.2

These two waves are π radians out of phase but have different amplitudes. Draw the output wave you would expect.



Tutorial 7.04 Standing Waves

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7.041 Stationary Waves

Sometimes waves appear to be standing still, *i.e.* the crests and the troughs appear to stay in the same place. We can see them in water, especially water surrounded by walls. We call them **standing waves** or **stationary waves**. Any kind of wave can form a standing wave.

Musical instruments depend on standing waves:

- In a string, for example guitar, pianoforte, violoncello.
- In a column of air, *e.g.* clarinet, tuba, organ.

Stationary waves are formed when two progressive waves are superposed:

- Equal frequency
- Nearly the same amplitude
- Same speed
- Travelling in **opposite directions**.

If we send an **incident** wave down a string, which is fixed at the end, the wave is reflected at the fixed end and undergoes a phase change of π radians or 180° (*Figure 40*). There is no phase change at the free end.

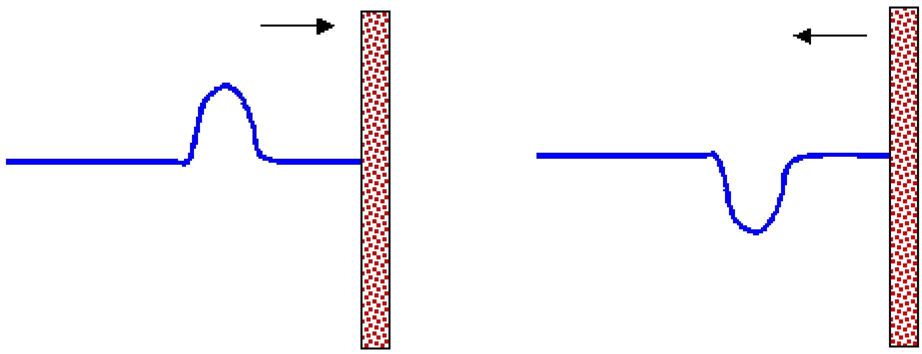


Figure 40 Phase change at the fixed end of a wave on a string.

7.042 Formation of Standing Waves

If we send a continuous stream of waves down the string, they are reflected, and a standing wave gets set up. The frequency will be the same, the amplitude very nearly the same and the speed will be the same. The directions are opposite. The phase change of π radians causes cancellation at the fixed end. This region of zero displacement is called a **node** (*Figure 41*).

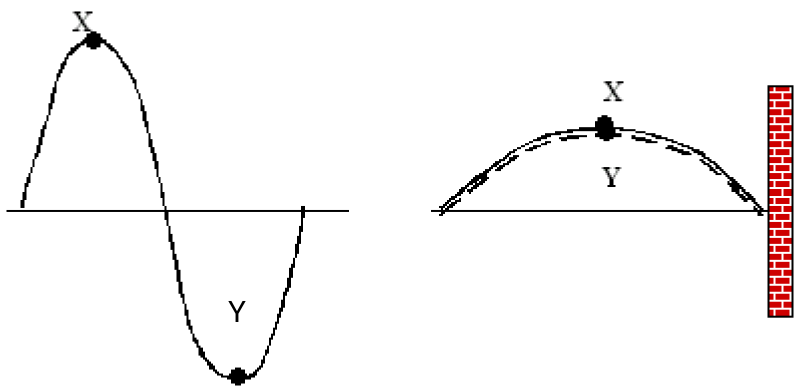


Figure 41 Two points in a wave that are p rad out of phase in a progressive wave form an antinode in standing wave

In a progressive wave, points **X** and **Y** would be in antiphase, π radians out of phase. However, because the wave is reflected, the phase is changed by π radians. So, they are now 2π radians out of phase, which means that they are in phase. Superposition is **constructive**. The amplitude is now at a maximum, and this is called an **antinode.**

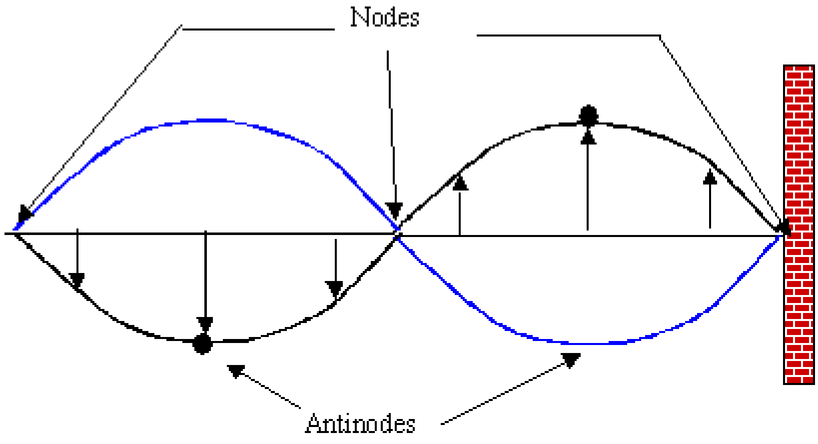


Figure 42 Standing (stationary) wave

Notice:

- All particles between nodes are **in phase**.
- All particles either side of a node are in **antiphase**.
- Each "loop" is **half** a wavelength ($\lambda/2$)

We can set up a standing wave using 3 cm wave apparatus (*Figure 43*).

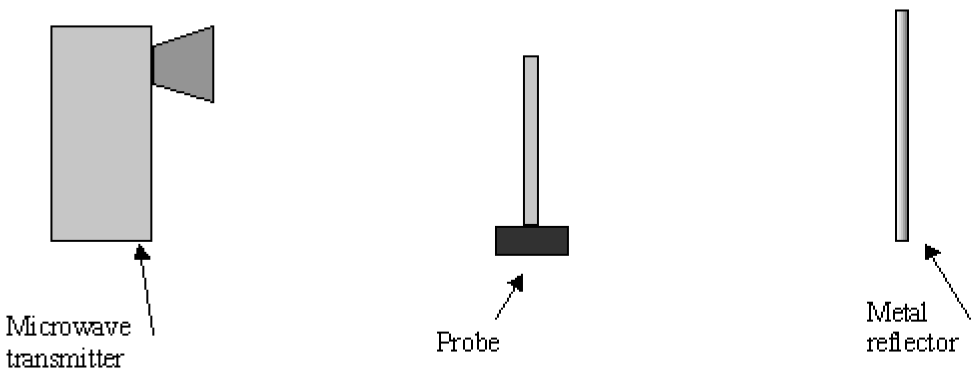


Figure 43 Standing waves using 3 cm microwaves

We move the probe between the transmitter and the reflector, and we detect maximum readings and minimum readings with the probe, which is connected to a microammeter. The maximum readings coincide with the antinodes, the minimum readings with the nodes.

A similar experiment can be done using sound waves.

7.043 Comparing Standing Waves with Progressive Waves

For all standing wave patterns, these two points are true:

1. The amplitude varies according to position from zero at a node, to maximum at an antinode. The amplitude of a given point is always the same.
2. The phase difference between two particles is zero if the points are between adjacent noted. It is 180° if they are either side of a node.

If the points are a separated by an even number of nodes, they are in phase (*Figure 44*).

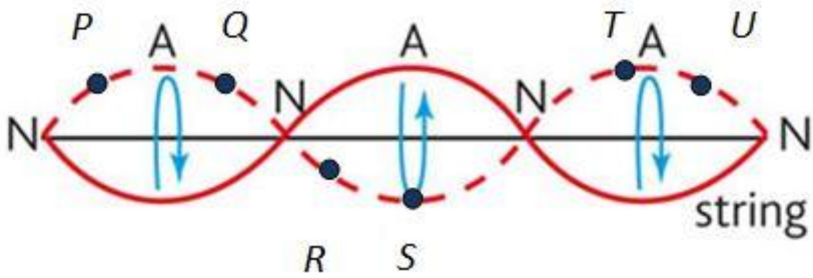


Figure 44 Phase relationships between different points on a standing wave

Therefore, *P* and *Q* are in **phase** with each other (as are *R* and *S,* and *T* and *U*).

P and *S* are in **antiphase**, but *P* is in **phase** with *U*.

Property	***Stationary Waves***	***Progressive Waves***
Frequency	All particles vibrate at the same frequency, except at the nodes where there is no vibration	All particles vibrate at the same frequency throughout the wave.
Amplitude	Amplitude varies between zero at the nodes and maximum at the antinodes. *S* will vibrate at a bigger amplitude to *R*.	The amplitude is the same for all particles.
Phase difference between two points.	Phase is $n\pi$ rad, where n is the number of nodes between the two points.	

7.044 Standing Waves in a String

Consider a taut string that is between two fixed points (*Figure 45*).

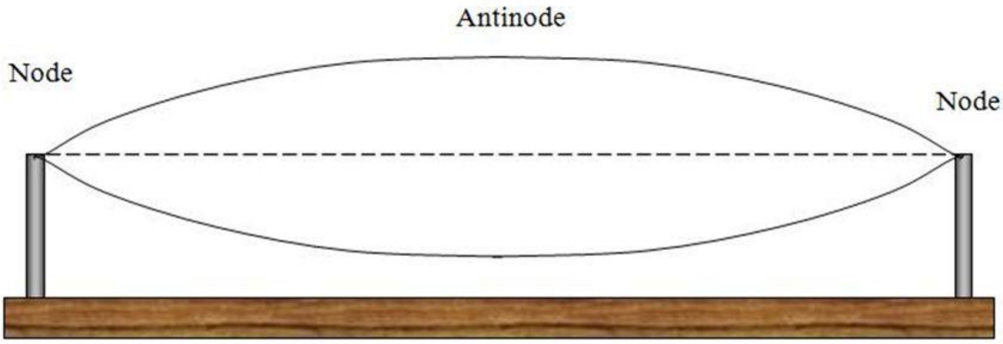


Figure 45 Standing wave in a string fixed at both ends

There are some rules for standing waves in strings:

- At the fixed end in a string, there is always a node.
- A string in a musical instrument is always fixed at both ends.
- Therefore, there is a node at each end.
- There is an antinode halfway down.
- At twice the fundamental frequency, there is a node in the middle of the string.

7.045 Frequency of a Stationary Wave

We can investigate standing wave in a string using equipment like this (*Figure 46*).

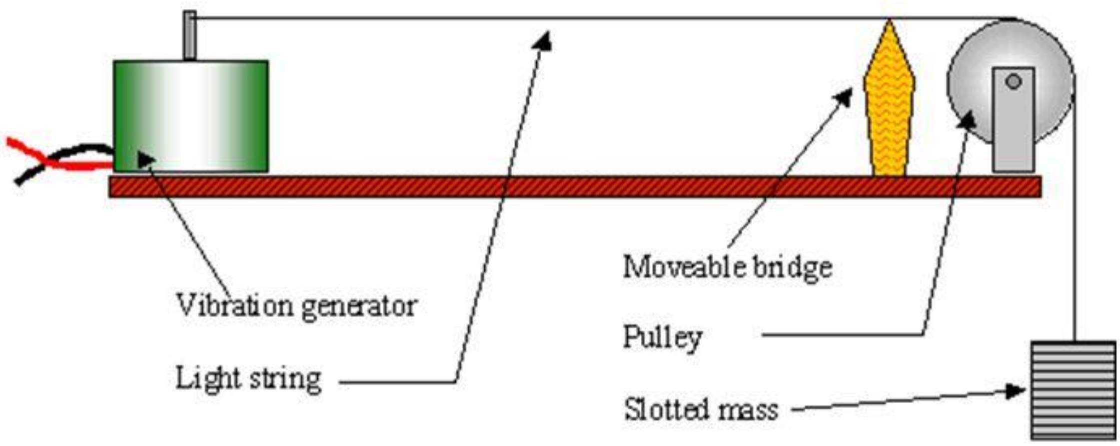


Figure 46 Melde's Apparatus

It is called **Melde's Apparatus**. If we start the frequency of the vibration at a low level, increasing it slowly, we see little of significance until at a certain value, a single large vibration loop is seen. This is due to **resonance** and is called the **fundamental frequency** or the **first harmonic**. The second harmonic has two vibration loops. It is twice the fundamental frequency. We will study resonance in Physics A-level.

In the AQA syllabus, the fundamental frequency is called the **first harmonic**, and the term *fundamental frequency* will not be used. The term *overtone* will not be used either. However, you will see these terms used in text books and other syllabuses.

The frequency at which resonance happens depends on:

- The tension
- The length
- The mass per unit length (how thick the string is).

7.046 The Speed of a Wave in a String (EDEXCEL syllabus).

The speed of a wave in a string depends on:

- The tension in the string.
- The mass per unit length.

The equation is:

$$v = \sqrt{\left(\frac{T}{\mu}\right)}$$

.... *Equation 19*

Where:

- v - speed (m s^{-1}).
- T - tension (N).
- μ - mass per unit length (kg m^{-1}).

The symbol μ is 'mu', a Greek lower-case letter 'm'.

We can show this graphically (*Figure 47*). The speed varies with direct proportionality with the **square root** of the tension.

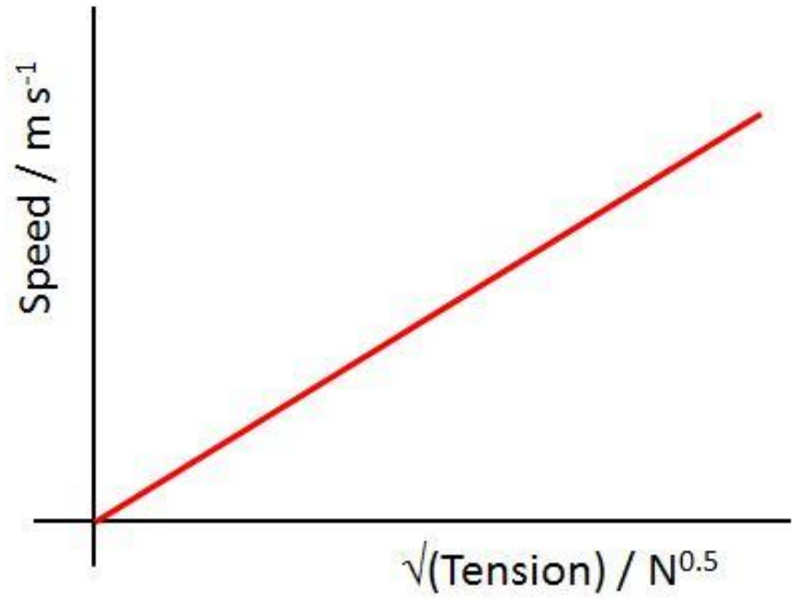


Figure 47 Proportionality between speed of a wave in a string and the square root of the tension

The speed is inversely proportional to the square root of the square root of the mass per unit length (*Figure 48*).

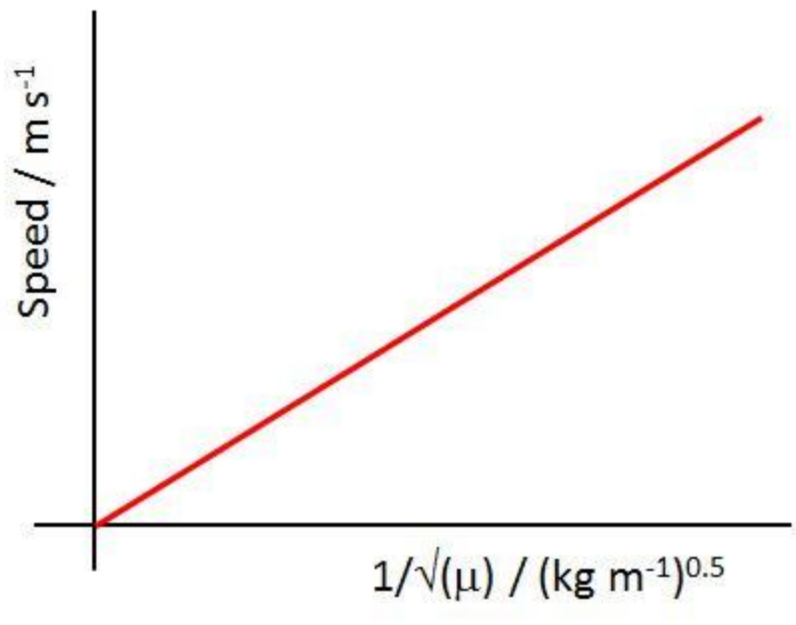


Figure 48 Proportionality between the speed of a wave in a string and 1/square root of the mass per unit length

Measuring the speed of a wave on a string is not at all easy to do directly. However, we can use the idea of the fundamental frequency in a stationary wave to decide what the wave speed is.

Consider a stationary wave that is formed by a wave travelling a v m s^{-1} along a taut string between two fixed points l m apart (*Figure 49*). The tension in the string is T N. The string has a mass per unit length of μ kg m^{-1}.

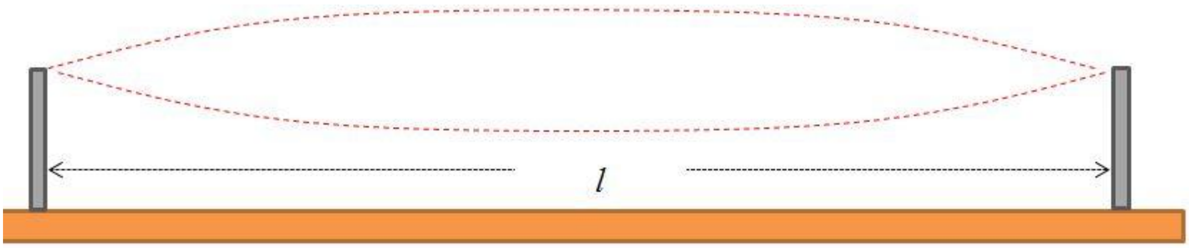


Figure 49 Standing wave between two fixed points

We know the wave equation:

$$v = f\lambda$$ .... *Equation 20*

Since this is a stationary wave at the first harmonic, we can say that the wavelength is $2l$. Therefore:

$$v = 2fl$$ .... *Equation 21*

We also know that the speed is related to the tension by the equation:

$$v = \sqrt{\left(\frac{T}{\mu}\right)}$$ .... *Equation 22*

It doesn't take a genius that in combining *Equations 21* and *22* we see that:

$$2fl = \sqrt{\left(\frac{T}{\mu}\right)}$$

.... *Equation 23*

So, we can rearrange *Equation 23* to make the **fundamental frequency**, f_0 the subject of the formula:

$$f_0 = \frac{1}{2l}\sqrt{\left(\frac{T}{\mu}\right)}$$

.... *Equation 24*

The Physics Codes are:

- f_0 - the fundamental frequency.
- l - the length of the string between the fixed points (m).
- T - the tension if the string (N).
- μ - the mass per unit length of the string (kg m^{-1}).

The symbol μ is 'mu', a Greek lower-case letter 'm'.

If you look inside a piano, the bass strings are not much longer than the middle note strings, but they are much thicker (*Figure 50*).

Figure 50 Bass strings of a piano (Picture: Frost Nova, Wikimedia Commons.)

<u>*Worked Example*</u>

A string is 2.3 m long between two fixed point. A length of 2.50 m has a mass 53.0 g. It is under a tension of 25.6 N. Calculate the first harmonic and give your answer to an appropriate number of significant figures.

Answer

Use:

$$f_0 = \frac{1}{2l}\sqrt{\left(\frac{T}{\mu}\right)}$$

Find the mass per unit length first:

$$\mu = 0.0530 \text{ kg} \div 2.50 \text{ m} = 0.0212 \text{ kg m}^{-1}$$

Put the numbers in:

$$f_0 = (1 \div (2 \times 2.3 \text{ m})) \times (25.6 \text{ N} \div 0.0212 \text{ kg m}^{-1})^{0.5} = 7.55 \text{ Hz} = \mathbf{7.6\ Hz} \text{ (2 s.f.)}$$

When rearranging the equation, it is easiest to square the equation:

$$f^2 = \frac{1}{4l^2}\left(\frac{T}{\mu}\right)$$

.... *Equation 25*

The mass per unit length is worked out from the total length of the string which may well be different from the length between the two fixed points.

Wire masses tend to be given in grams. Make sure you convert the mass in grams to kilograms. (You do know how to do this, don't you?)

7.048 Energy and Power in a Wave on a String (A-level Extension)

Consider a wave of amplitude A m and wave length l m travelling on a string of mass per unit length μ kg m^{-1} at a speed of v m s^{-1} at a frequency of f Hz.

The frequency is represented by the relationship:

$$\omega = 2\pi f$$

.... *Equation 26*

The energy associated with one wavelength is given by:

$$E_\lambda = \frac{1}{2}\mu\omega^2 A^2 \lambda$$

.... *Equation 27*

This is consistent with the relationship we met in Tutorial 7.040

$$E = kA^2$$

.... *Equation 28*

The constant is:

$$k = \frac{1}{2}\mu\omega^2\lambda$$

.... *Equation 29*

We can check the **consistency** of this by looking at the **units**. In tutorial 1, we saw that the units for k were J m^{-2}. In base units:

1 J = 1 kg m^{2} s^{-2}

The units for k are therefore:

J m^{-2} = kg m^{2} s^{-2} × m^{-2} = kg s^{-2}

The units for k in the equation above are:

kg m^{-1} × (rad^{2}) s^{-2} × m

Since rad is a dimensionless unit, we can say that k has units of:

kg s^{-2}

Therefore, the units of k are **consistent in both cases**.

We know that power is related to energy by:

power (W) = energy (J) ÷ time (s)

In waves, we know that each wave takes a time period T. Each wave has an energy per wave of E_λ. So, we can write an equation for the power of the wave:

$$P = \frac{E_\lambda}{T}$$

.... *Equation 30*

So, we can write:

$$P = \frac{1}{2}\mu\omega^2 A^2 \frac{\lambda}{T}$$

.... *Equation 31*

Now we know that:

wave speed (m s^{-1}) = wavelength (m) ÷ Time period (s)

So, we can write from *Equation 31*:

$$P = \frac{1}{2}\mu\omega^2 A^2 v$$

.... *Equation 32*

We can go one step further since we know that:

$$v = \sqrt{\left(\frac{T}{\mu}\right)}$$

.... *Equation 33*

So, we can substitute *Equation 33* into *Equation 32* to give:

$$P = \frac{1}{2}\mu\omega^2 A^2 \sqrt{\left(\frac{T}{\mu}\right)}$$

.... *Equation 34*

It tidies up to give:

$$P = \frac{1}{2}\omega^2 A^2 \sqrt{(T\mu)}$$

.... *Equation 35*

This is not on the syllabus, but you might be asked to derive it using equations that you have been given.

7.04.9 Required Practical

We can carry out an experiment using a vibration generator and a string under load (*Figure 51*).

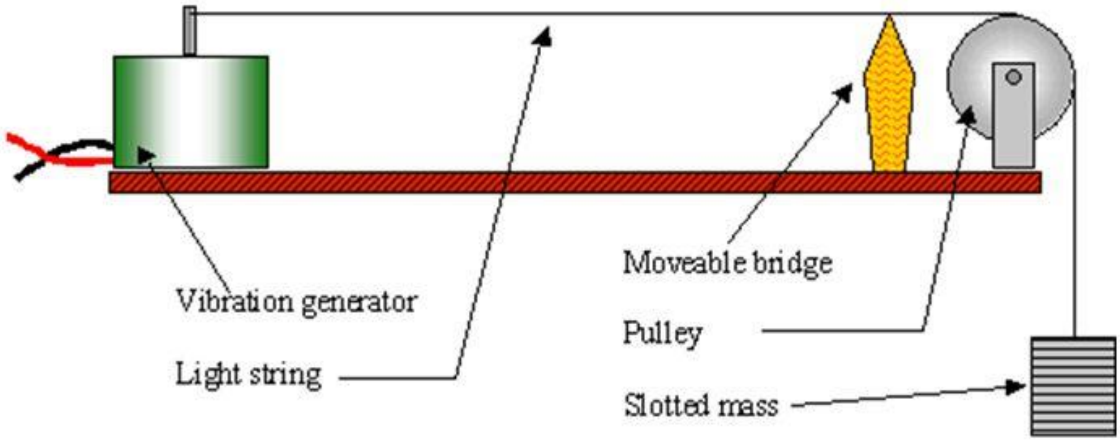


Figure 51Measuring how fundamental frequency varies under different conditions

We can do the experiment in two different ways:

- Measure the frequency and measure the length of the string to give a first harmonic. The tension is kept the same.
- Keeping the length the same, the frequency and tension are changed.

The graphs are like this (*Figures 51* and *52*).

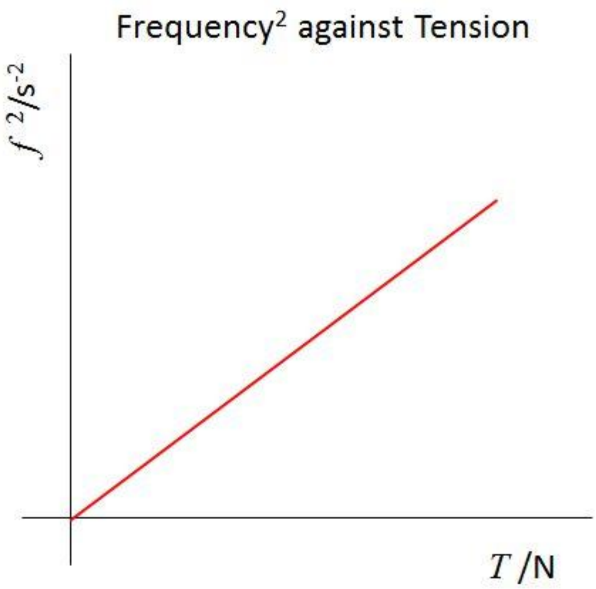


Figure 52 Graph of square of frequency against tension

This graph shows that frequency^2 is directly proportional to the tension.

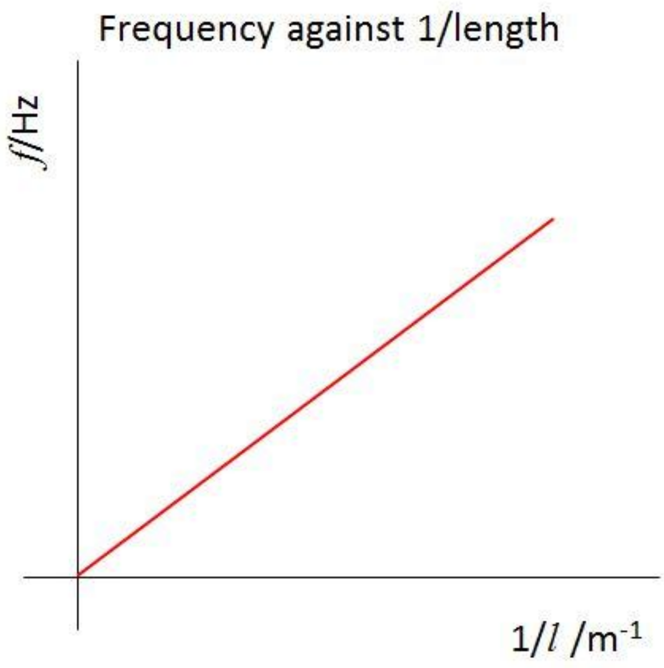


Figure 53 Graph of frequency against 1/length

This graph shows the frequency is inversely proportional to the length of the string.

If you can, use a **frequency meter** to get the value for the frequency. Most signal generators in school or college labs are not that well calibrated. Alternatively, you should check the calibration of the signal generator using a CRO. If you haven't done the CRO, your tutor will show you how to do this.

Tutorial 7.04 Questions

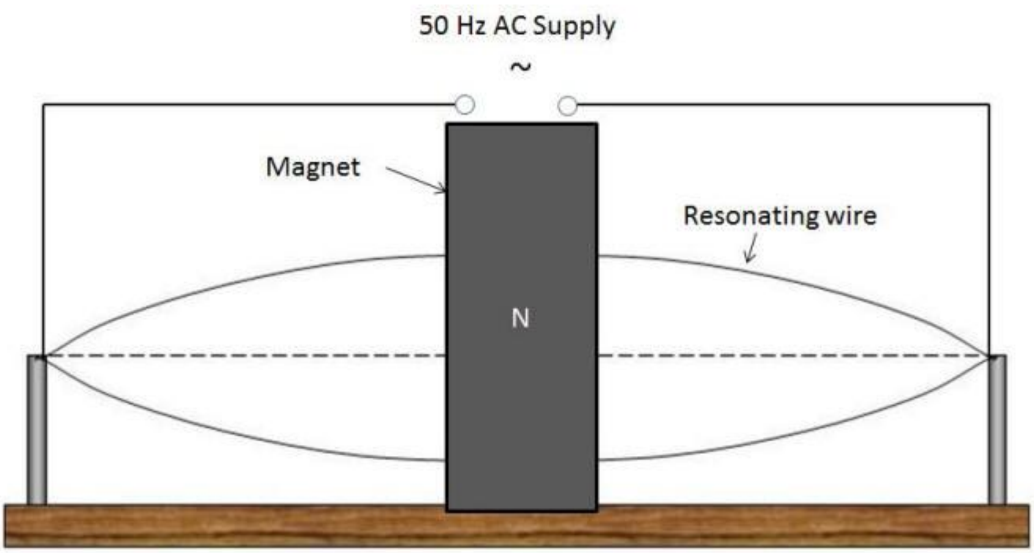
7.04.1

What are the conditions needed for a standing wave?

7.04.2

What is meant by a node and an antinode? A standing wave loop in a string is 66 cm long. What is the wavelength in metres?

7.04.3



A length of wire has a mass of 1.25 g for a length of 1.60 m.

In an experiment to measure mains frequency that is shown above, it is required to fit between two fixed points 1.15 m apart and is placed between the poles of a magnet. It is found to resonate at a first harmonic with a frequency of 50.0 Hz.

(a) Calculate the tension needed.

(b) Calculate the speed of the waves in this wire.

7.04.4

(Extension) In the experiment in Question 7.04.3, the amplitude was found to be 20 mm.

(a) Using your answers to Question 7.04.3, work out the power required.

(b) The apparatus is connected to power supply that gives an AC potential difference of 2.5 V RMS. What is the current?

Give your answers to an appropriate number of significant figures.

Tutorial 7.05 Making Music

Extension, Scottish Advanced Higher

Contents

The artist in the picture below (*Figure 54*) is the composer and musician Mike Oldfield in a performance on 1st December 2006 (rather a long time ago for many readers). Oldfield made a name for himself as a young composer in the nineteen-seventies, with albums such as *Tubular Bells*.

Figure 54 Mike Oldfield (Alexander Schweigert, Wikimedia commons)

Musical instruments make their sounds by standing waves. Standing waves can be set up by:

- plucking a tightened string, or scraping it with a bow, or striking it with a hammer.
- blowing a raspberry (an ***embouchure***) to a mouthpiece.
- vibrating air using a reed or a whistle.

Different notes can be made by changing the length of the string, or the length of the pipe. The volume is changed by altering the amplitude of the wave.

Musical notes have the following properties:

- The **frequency** or **pitch**.
- The **amplitude** or **volume**.
- The **quality**, or **timbre**.

The pitch is NOT the volume.

7.051 Stringed Instruments

We can show standing waves on a string with **Melde's Apparatus** (*Figure 55*).

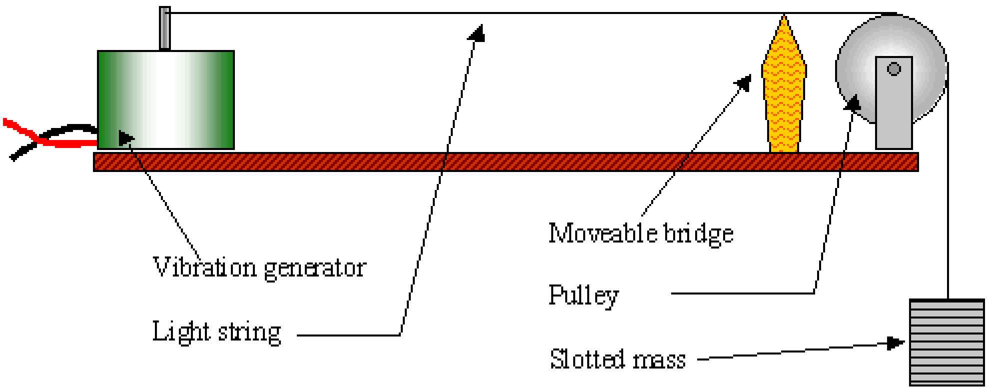


Figure 55 Melde's Apparatus

If we start the frequency of the vibration at a low level, increasing it slowly, we see little of significance until at a certain value, a single large vibration loop is seen. This is due to **resonance** and is called the **fundamental frequency** or the **first harmonic**. The **second harmonic** has two vibration loops. It is twice the fundamental frequency. We will study resonance in A-level Physics. We have discussed the fundamental frequency (first harmonic) in the previous tutorial.

The frequency at which resonance happens depends on:

- The tension
- The length
- The mass per unit length (how thick the string is).

Although this is NOT on the AQA AS level syllabus, the fundamental frequency can be calculated using the formula:

$$f_0 = \frac{1}{2\pi L}\sqrt{\frac{T}{\mu}}$$

.... *Equation 36*

The Physics Codes are:

- f_0 - the fundamental frequency.
- L - the length of the string (m).
- T - the tension if the string (N).
- μ - the mass per unit length of the string (kg m^{-1}).

We have done this in Tutorial 7.04.

If you look inside a piano, the bass strings are not much longer than the middle note strings, but they are much thicker (*Figure 56*).

Figure 56 Bass strings in a piano (Picture: Frost Nova, Wikimedia Commons.)

A string at the fundamental frequency looks like this (*Figure 57*):

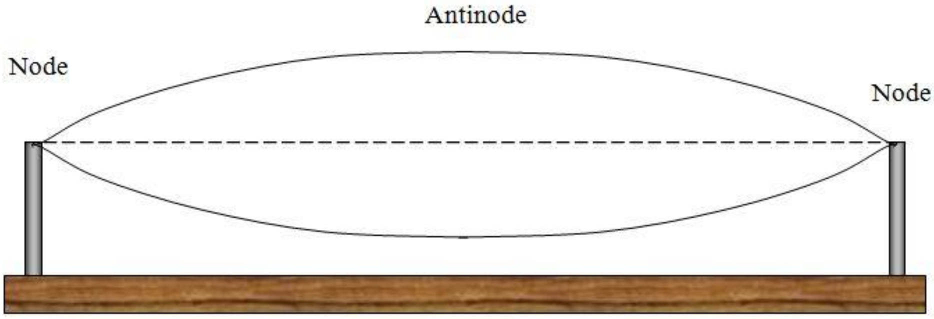


Figure 57 String at fundamental frequency

You can see that it forms one resonance loop, which is 1/2 of one wavelength ($\lambda/2$). If we increase the frequency, no pattern is observed until we reach a frequency of twice the fundamental frequency ($2f_0$). The distance between the two fixed ends is one wavelength (*Figure 58*). Each resonance loop is 1/2 wavelength. This is called the **second harmonic**, or **first overtone**.

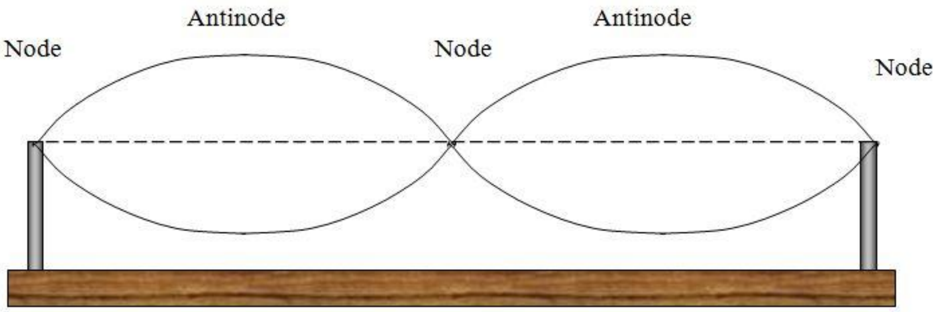


Figure 58 Second Harmonic

And if we increase the number of resonance loops to 3, we have a frequency of $3f_0$. This is the **third harmonic,** or **second overtone** (*Figure 59*).

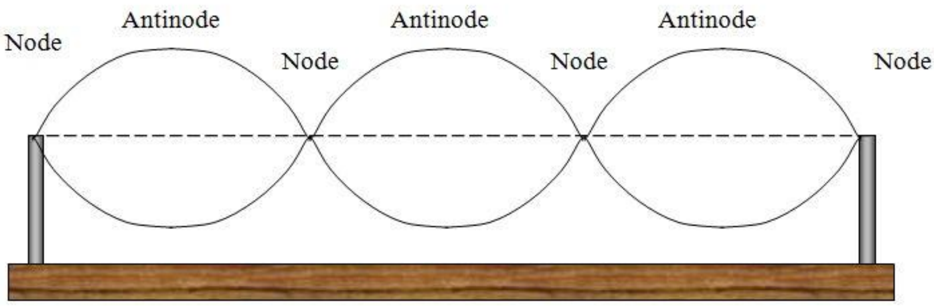


Figure 59 Third harmonic

Each whole number change in fundamental frequency represents a change in note of 1 octave. An octave is formed by 8 notes on the scale, shown by the keys on a piano (*Figure 60*):

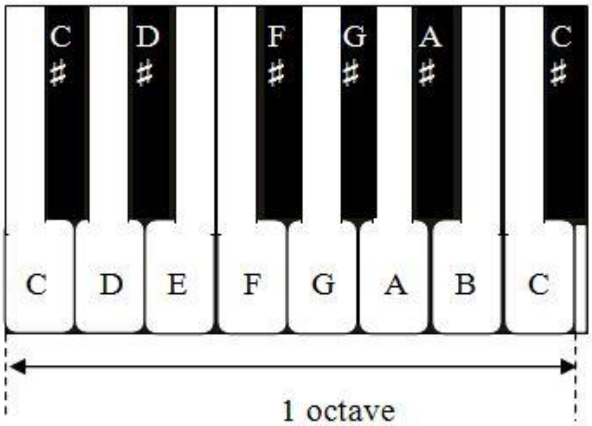


Figure 60 Octave on a piano

The different notes are achieved by having strings (or pipes) of different lengths.

Middle C is, as the name suggests, in the middle of a piano keyboard.

7.052 Frequencies (Extension)

In an orchestra, all instruments have to be tuned to a particular frequency, otherwise the resulting din would be unpleasant to listen to. By convention, the note **A** (above Middle C) is sounded at a frequency of 440 Hz. The table below shows the frequencies that are used by instruments tuned to **International Pitch Notation**. The table shows the notes in the octave between Middle C and the C above middle C.

Note	*Solfège*	*Frequency / Hz*
C_4	Do (Ut)	261.626
$C\#_4$	Do#	277.183
D_4	Re	293.665
$D\#_4$	Re#	311.127
E_4	Mi	329.628
F_4	Fa	349.228
$F\#_4$	Fa#	369.994
G_4	Sol	391.995
$G\#_4$	Sol#	415.305
A_4	Fa	440
$A\#_4$	Fa#	466.164
B_4	Si	493.883
C_5	Do	523.251

Notes:

- The Solfège-do system of naming notes is used in many countries such as France, Spain, and Italy. It was invented by Guido of Arezzo (991 - 1033) and developed by other musical theorists. The names came from the first syllables to lines from a hymn in Latin to St John the Baptist (**Ut** queant laxis, **Re**sonare fibris, **Mi**ragestorum, **Fa**muli tuorum…).
- In Britain, Germany, and Sweden the note letters are used.
- In the table, I have missed out the flat notes. The note G-flat (Gb) is the same note on the piano as F-sharp (F#).
- In some musical scores, notes like E and B are made sharp. E-sharp is played on the note F and B-sharp is played on the note C. Similarly, F-flat is played on the note E.
- The subscript number 4 shows that it's the fourth octave on the piano (*Figure 61*).

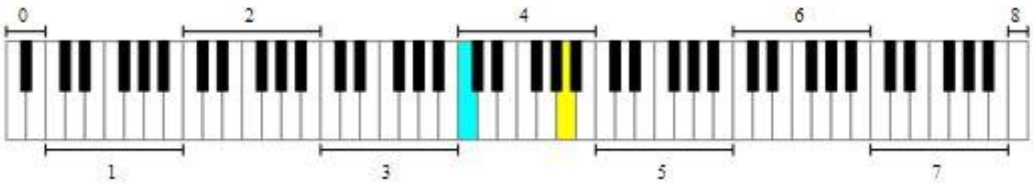


Figure 61 Octaves on a piano keyboard (Image from Wikimedia Commons. Author: Alwaysangry)

The frequency **doubles** for every **octave**. The note A below middle C has a frequency of 220 Hz. There is a formula that is used to work out the frequency. Note number 1 on a piano keyboard is A_0. Note number 88 is C_8. For the nth key:

$$f(n) = 2^{\frac{n-49}{12}} \times 440 \text{ Hz}$$

.... *Equation 37*

Worked example Calculate the frequency of note number 32 on the piano.
Answer $f(n) = 2^{32-49/12} \times 440 \text{ Hz} = 2^{-1.417} \times 440 \text{ Hz} = 164.814 \text{ Hz}$ This is Note E in the third octave (E_3).

This formula is not on the syllabus, but it may come up in one of those question that introduces a new concept and assesses how you tackle it.

Musical scores for most instruments use the same key as the piano. However, some instruments like the clarinet use a score that is written in a different key. If the score for a clarinet is written in C, the piano score has to be written in B-flat. If the piano score was written in C, the two instrument would be out of tune.

7.053 Wind Instruments

Wind instruments make sounds by making **columns of air** resonate. A church organ such as the instrument in this picture (*Figure 62*) has many hundreds of pipes that give the instrument the ability to make a wide range of different sounds.

Figure 62 A church organ

Air columns as in an organ make **longitudinal** standing waves, which we can show with a **Kundt's tube** (*Figure 63*). (August Kundt (1839 - 1894))

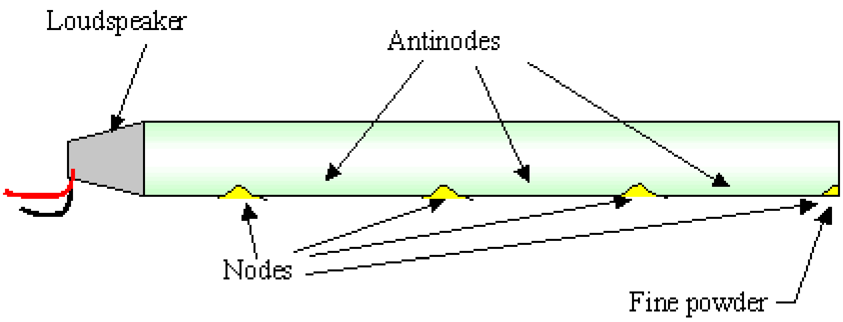


Figure 63 Kundt's tube

Be careful how you pronounce "Kundt's"; it is best said "*Kunst*".

We should note that:

- The wave is **longitudinal** so that all the particles vibrate **parallel** to the tube.
- The amplitude is at a **maximum** at the open end of the tube, so there must be an **antinode**.
- The amplitude at the closed end is **zero**; there is a **node**.
- All molecules between nodes vibrate **in phase**.
- All molecules either side of a node vibrate in **antiphase**.
- Adjacent nodes are **half a wavelength** apart.

In a church organ a motor blows air into a reservoir, which in turn supplies a **chest** where the valves that sound the notes are. The keys (which are like those on a piano) operate air valves to allow air into the pipe. A whistle arrangement sets the air vibrating at the fundamental frequency. Large instruments will have three keyboards, so that the organist can mix the sounds that the instrument can make. There is also a keyboard operated by the feet, which plays the bass notes. The organist has a variety of stops which enables her to choose the sounds that she wants for the music she is going to play.

In the old days (and in churches where there is no electricity supply) somebody had the task of pumping the air into the organ. If the music got loud, the person doing the pumping would have to pump hard, or the instrument would run out of puff, and the music would fade away. The classical composer, Felix Mendelssohn, played the organ at St Paul's Cathedral in London. His playing was so enjoyed by the congregations after church services that they stayed on to listen. But the vergers wanted them to get out of the church so they could lock it up. So, the people who were pumping the organ were told to stop, leaving the young Mendelssohn playing on silent keys.

At fundamental frequency, a **closed organ pipe** has half a vibration loop. Note that we are representing the longitudinal wave graphically (*Figure 64*).

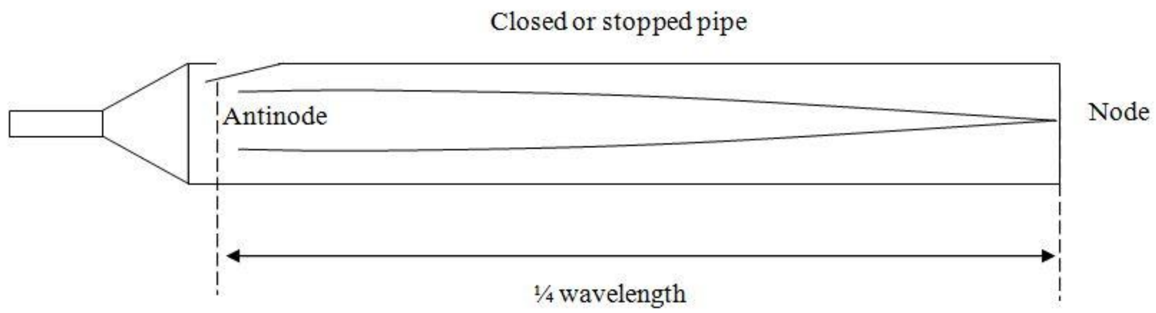


Figure 64 Fundamental frequency in a closed organ pipe

Since this is ¼ of a wavelength, the organ pipe sounds a note whose wavelength is 4 times its length. The antinode is formed by air passing a whistle arrangement. Very deep bass notes have large, stopped pipes.

A couple of rules when looking at air columns:

- At the open end, there is always an **antinode**.
- At the closed end, there is always a **node**.

For the closed pipe, the pattern for the next harmonic looks like this (*Figure* 65):

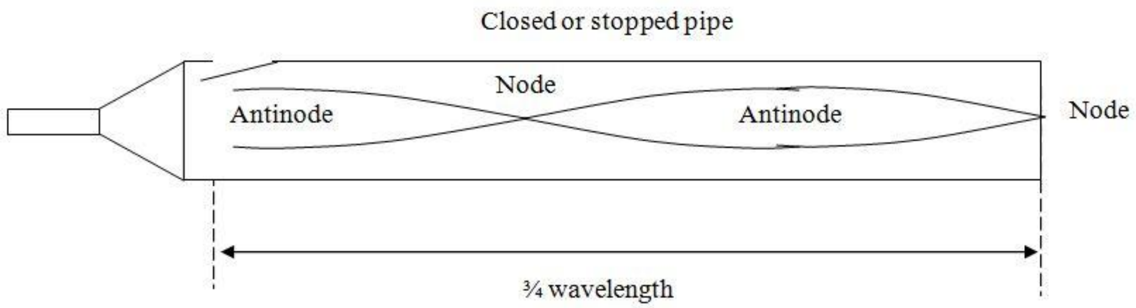


Figure 65 Second harmonic in a closed pipe

This gives a second harmonic in which the frequency is $3f_0$. The next harmonic is $5f_0$.

Organs have **open** pipes as well. The fundamental wave pattern looks like this (*Figure 66)*.

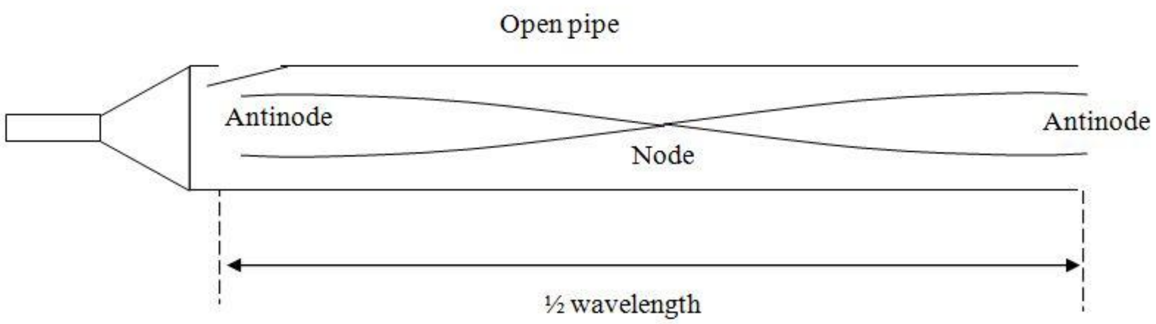


Figure 66 Fundamental frequency of an open pipe

In this case the wavelength of the note is twice the length of the pipe.

The next harmonic happens at $2f_0$ (*Figure 67*).

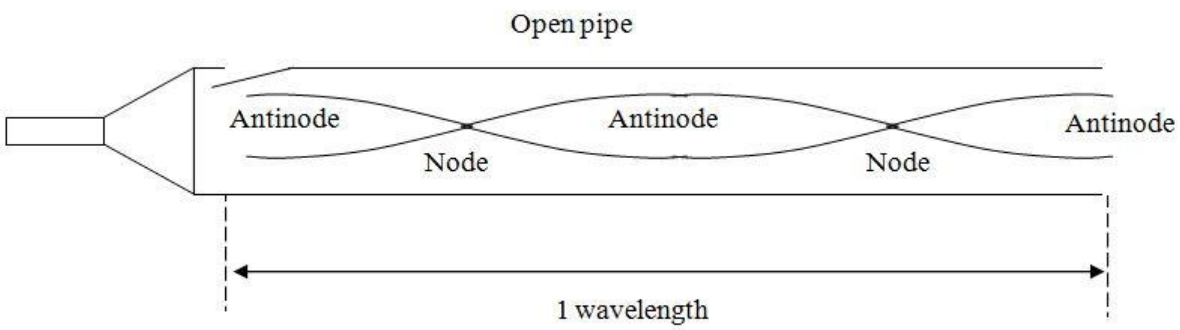


Figure 67 Second harmonic in an open pipe

Them harmonics work like those on a string, i.e. $2f_0$, $3f_0$, $4f_0$, and so on.

This has important implications for wind instruments. The picture (*Figure 68*) shows a trumpeter blowing into his **embouchure**.

Figure 68 A trumpeter's embouchure

A brass instrument like a trombone has an antinode made by the player pursing his lips and vibrating them (an *embouchure*). The mouthpiece of the trumpet is called an **embouchure** as well. There is an antinode at the bell of the instrument, and a node half way down (*Figure 69*).

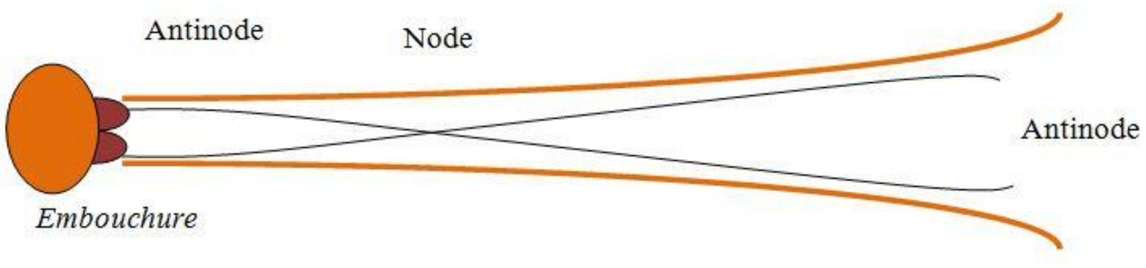


Figure 69 An antinode if formed at the embouchure of a brass instrument

The fundamental frequency can be changed by altering the length of the tube. In a trombone, the player moves part of the tube in and out. A trumpeter changes the length by pressing in valves. The trumpeter can go up octaves by blowing harder, setting up standing waves in the second or third harmonic. However, the trumpet cannot play bass notes below its lowest fundamental frequency.

7.054 Investigating Standing Sound Waves

We can do an experiment to work with standing sound waves using apparatus like this, as shown in *Figure 70*.

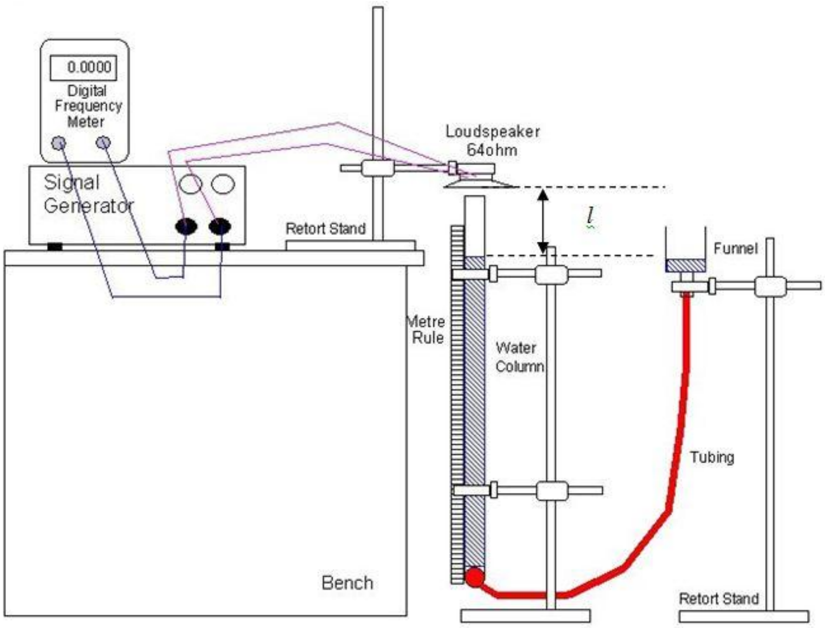


Figure 70 Investigating standing sound waves

We carry out the experiment using the following method:

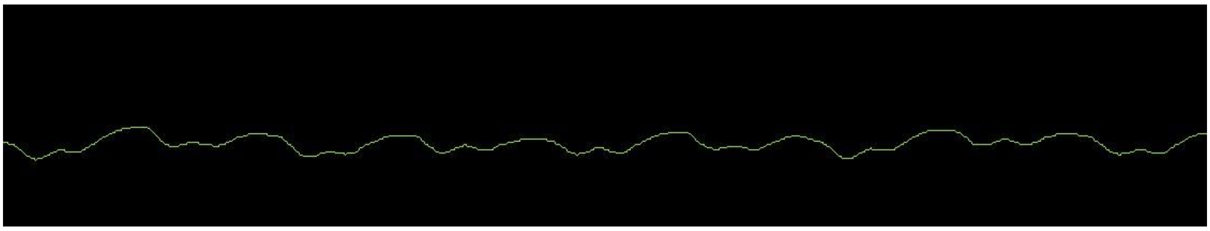
- Adjust the water level so that l is about 25 cm.
- Switch on the signal generator and set the frequency is about 250 Hz.
- Adjust the volume control so that the tone can just be heard.
- Increase the frequency carefully until there is a sharp increase in volume.

We note the frequency at which the increase of volume occurs. We know that there is a node at the water surface and an antinode at the end of the glass tube.

The next maximum amplitude will be at a frequency of 3 time the fundamental frequency.

7.055 Quality of Sound

The principles we have discussed have modelled the sound as sine waves (or pure tones). These are very dull to listen to, and musical instruments make much more complex sound patterns, which give them their distinctive sounds. An organ playing the note D will sound different to a trumpet playing the same note. This is often referred to as the **quality** (or *timbre*) of the sound. Playing a musical instrument into a CRO will show this. You can set the *visualisation* function on your PC music player to *scope* and see the music as it would be displayed on the CRO.



Complex waveforms like those in musical instruments are discussed further in a later tutorial.

Music is a truly international language, that is there to be enjoyed by everyone. It acts at a far deeper psychological level than words. You do not have to be a professional musician to write or make music. The classical composer, Alexander Borodin, was little known as a composer when he was alive; he was the Chemistry professor at the University of St Petersburg, whose research gave us much of organic chemistry as we understand it today. Nowadays, he is better known as a composer than a chemist, although organic chemists use his results every day.

On the other hand, you would not want to hear my piano-playing. I do well if I get 80 % of the notes right (and not necessarily in the right order, or the right time). There is an orchestra for those who are as musically challenged as I am.

Music can unite but also divide. Someone's taste in music may disgust somebody else. (And for most people at the age of about 35, something dreadful seems to happen to pop music.) Another of my favourite composers at the time the picture below was taken was Rick Wakeman. He appeared on the BBC's series *Grumpy Old Men*. He was also the resident grumpy for *Watchdog on* BBC 1.

Figure 71 Playing recorded music. A long time ago...

The record on the record deck is one of Mike Oldfield's compositions, *Ommadawn*, I think. His boxed set is certainly there; I still have it.

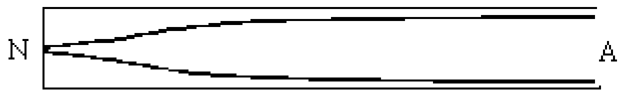
Tutorial 7.05 Questions

7.05.1

How do you think these principles apply to musical instruments? Explain how the instrument can be tuned.

7.05.2

The diagram shows a closed organ pipe being played at its fundamental frequency.



What do you think the harmonics will be like? What happens to the frequency? Draw diagrams to illustrate your answer.

7.05.3

What happens if the pipe is open at both ends at fundamental frequency? What harmonics do you get? Draw a diagram to illustrate your answer.

7.05.4

In music, the frequency of a note doubles if you go up an octave. Some organs have very deep bass notes. A deep bass note A has a frequency of 22.5 Hz.

(a) How many octaves is this below the note A of concert pitch 440 Hz.

(b) If sound has a speed of 340 m/s, what is the wavelength?

(c) What is the minimum size of pipe needed? What form should it take? Explain your answer.

7.05.5

The length of air column in the experiment on Page 76 was found to be 27 cm when the increase in volume occurred. What was the frequency, assuming the speed of air to be 340 m s^{-1}?

3. Wave Behaviour of Light

Tutorial 7.06 Reflection, Refraction, and Optical Fibres

All Syllabi

Contents

We have seen in Particle Physics how light behaves as a particle and how particles behave as waves. In this tutorial, we are going to look at how light shows **wave behaviour**. All other waves behave in the same way.

Some Revision

In this section we will consider light to be a wave carrying energy that travels in straight lines at a speed of 3.0×10^8 m s^{-1} in air. When light hits an object, it is (*Figure 72*):

- Reflected
- Transmitted
- Absorbed.

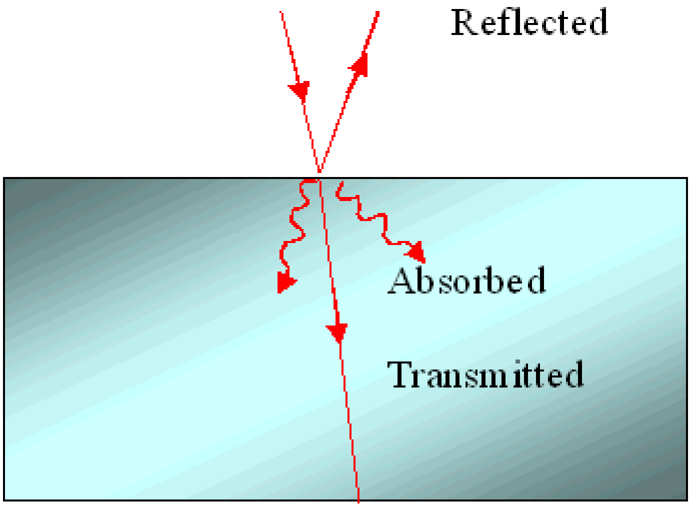


Figure 72 Revising refraction

7.061 Reflection

You should be familiar with reflection which you did in early secondary school. All reflection depends on the smoothness of the surface. If the surface is smooth, then parallel rays are reflected parallel (**specular** reflection). If the surface is rough, then parallel rays are **scattered**. However, each individual ray obeys the **law of reflection**.

High quality mirrors are silvered on the front to prevent multiple images. Here is a ray of light striking a mirror at an angle (*Figure 73*).

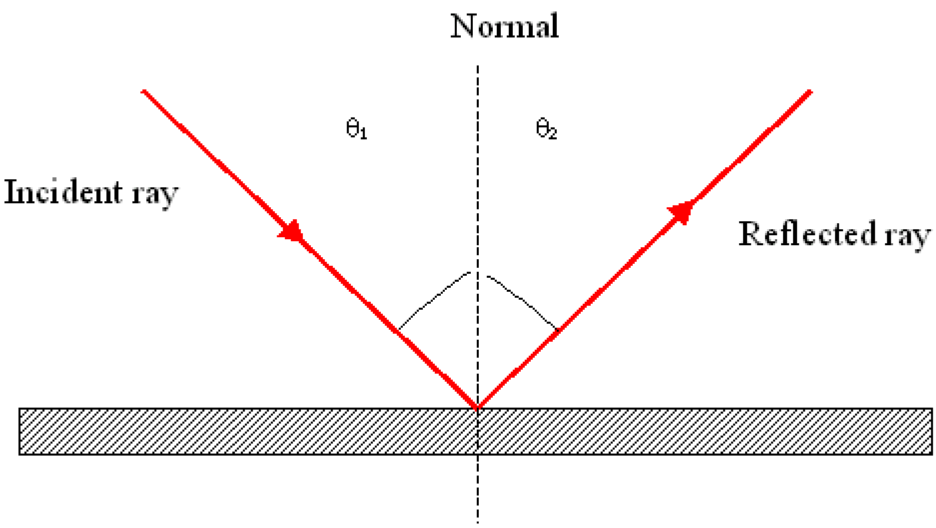


Figure 73 Reflection of light

We observe the following:

- The angle of **incidence** = angle of **reflection**, $\theta_1 = \theta_2$
- The reflected ray is in the **same plane** as the incident ray.
- All angles are measured from the **normal**, a line running at 90° to the surface.

The **image** in a mirror has these features:

- It is the same distance behind the mirror line as the object is in front.
- It is the same size as the object.
- It is the right way up (**erect**).
- It is **laterally inverted**, i.e. left and right swapped over.
- It is **virtual**, meaning that the rays cannot be projected onto a screen. The image is not really behind the mirror; it just *appears* to be.

Curved mirrors obey the same rules of reflection as flat (**plane**) mirrors do.

Reflections can give a lot of atmosphere to a photograph - what an excuse to share one of my favourites! (*Figure 74*).

Figure 74 Reflections in a landscape

7.062 Curved Mirrors

These notes in this section are for students doing the Irish syllabus.

Before we look at the reflecting telescope, we should have a quick revision at curved mirrors. There are two kinds:

- The concave mirror. You may well have one in the shaving mirror in the bathroom.
- The convex mirror. You will see these used as security mirrors in shops.

A **concave** mirror brings parallel rays of light together (*Figure 75*).

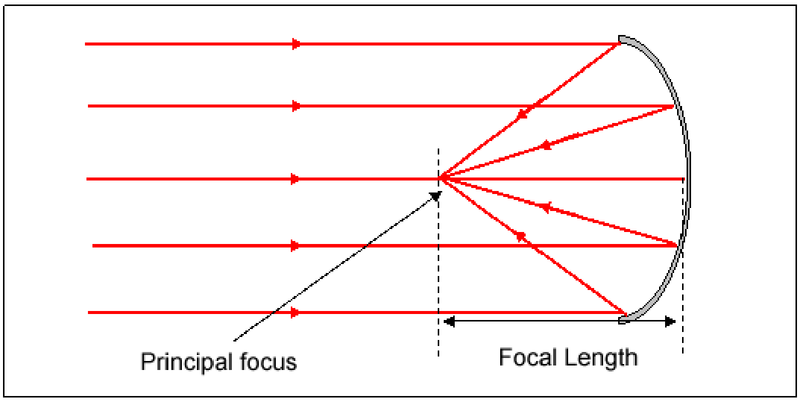


Figure 75 Concave mirror

Each ray obeys the **Law of Reflection**. You can see that the rays come together or **converge**.

Note how the shape of the mirror brings all the rays to a single point called the **principal focus**. The distance between the principal focus and the surface of the mirror is called the **focal length**.

If the object is close up to the mirror, it appears the right way up (**upright** or **erect**) and is **magnified** (made bigger). If it's further away the image is upside down (**inverted**) and **diminished** (made smaller).

Other waves can be reflected by a concave mirror. A satellite dish is a concave mirror to reflect microwave waves onto an antenna. There was a device produced after the First World War to focus sound waves of incoming aircraft to give early warning of their presence. And it worked.

<u>Convex Mirror</u>

A **convex** mirror reflects light rays outwards as shown in the diagram (*Figure 76*).

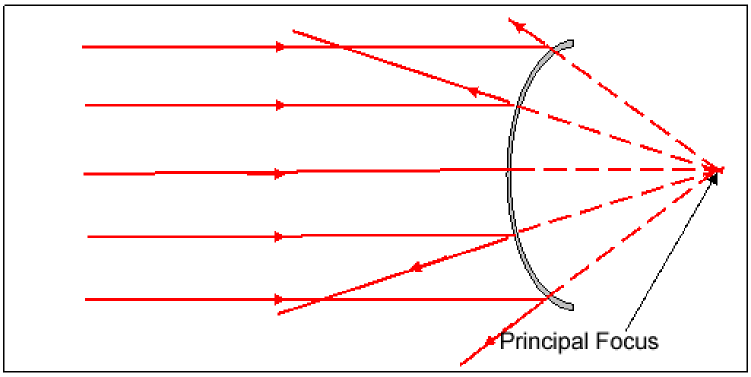


Figure 76 Convex mirror

If we extend the rays behind the mirror, we see that they meet at a **principal focus**. The image is **virtual**, **upright**, and **diminished**.

We can draw the ray diagrams with accurate drawing, but we need to know the shape of the mirror. It is easiest if the mirror is considered to be **spherical**, rather than parabolic. The focal length of a spherical mirror is half the radius.

$$f = \frac{r}{2}$$

.... Equation 38

You will, of course, need a compass to draw the mirror line on your graph paper. Remember that the angle of incidence with the normal line to the surface of the curved mirror is the same as the angle of reflection from the normal line. (Drawing the normal line to the curved surface is easier said than done.)

The **curved mirror formula** is of similar form to the lens formula:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

.... Equation 39

[f - focal length (m); u - object distance (m); v - image distance (m)]

The **sign conventions** are:

- f is positive for a concave mirror.
- f is negative for a convex mirror.
- v is positive for a real image.
- v is negative for a virtual image.

The **magnification** (ratio of image height to object height) is given by:

$$M = \frac{v}{u}$$

.... Equation 40

Worked example

A concave mirror is spherical with a radius of 30 cm. A light bulb is 3.0 cm high and is placed 12.0 cm from the mirror on the principal axis.

What is the image distance?

What is magnification?

What is the image height?

Is the image real or virtual?

Answer

Focal length needs to be worked out:

$$f = 30 \text{ cm} \div 2 = 15 \text{ cm}$$

Since it's a concave mirror, the focal length is positive.

Now we use the equation:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$(15 \text{ cm})^{-1} = (12 \text{ cm})^{-1} + v^{-1}$$

$$v^{-1} = 0.0667 \text{ cm}^{-1} - 0.0833 \text{ cm}^{-1} = -0.0167 \text{ cm}^{-1}$$

$$v = \mathbf{\underline{-60 \text{ cm}}}$$

Magnification Equation:

$$M = \frac{v}{u}$$

$$M = 60 \text{ cm} \div 12 \text{ cm} = 5.0$$

Image height = 3.0 cm × 5.0 = **15 cm** (ignoring the minus sign)

The image is virtual because the sign for the image distance is negative. It is the right way up.

(Strictly speaking the magnification equation is:

$$M = -v/u = - -60 \text{ cm}/ 12 \text{ cm} = \mathbf{\underline{+5.0}})$$

If we place the object on the other side of the principal focus (i.e. $u > f$) the image is real, magnified, and inverted.

Now we will do the same for a convex mirror.

<u>*Worked example*</u>

A convex mirror is spherical with a radius of 30 cm. A light bulb is 3.0 cm high and is placed 12.0 cm from the mirror on the principal axis.

What is the image distance?

What is magnification?

What is the image height?

Is the image real or virtual?

<u>*Answer*</u>

Focal length needs to be worked out:

$$f = -30 \text{ cm} \div 2 = -\mathbf{15\ cm}$$

Since it's a convex mirror, the focal length is negative.

Now we use the equation:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$-(15 \text{ cm})^{-1} = (12 \text{ cm})^{-1} + v^{-1}$$

$$v^{-1} = -0.0667 \text{ cm}^{-1} - 0.0833 \text{ cm}^{-1} = -0.15 \text{ cm}^{-1}$$

$$v = \mathbf{-6.67\ cm}$$

Magnification Equation:

$$M = \frac{v}{u}$$

$$M = 6.67 \text{ cm} \div 12 \text{ cm} = \mathbf{0.556}$$

Image height = 3.0 cm × 0.556 = **<u>1.67</u>** cm (ignoring the minus sign)

The image is virtual because the sign for the image distance is negative. It is the right way up.

(Strictly speaking the magnification equation is

$M = -v/u = - -6.67$ cm/ 12 cm = +**<u>0.556</u>**)

7.063 Refraction

Light rays are bent when they travel from a medium of one **optical density** into another, for example from air to glass. In air light travels at 3.0 × 10^8 m s^{-1}, while in glass its speed is 2.0 × 10^8 m s^{-1}. We say that glass in an **optically denser** medium than air. We can see that the direction of the ray is abruptly changed, or **deviated** as it passes the boundary (*Figure 77*):

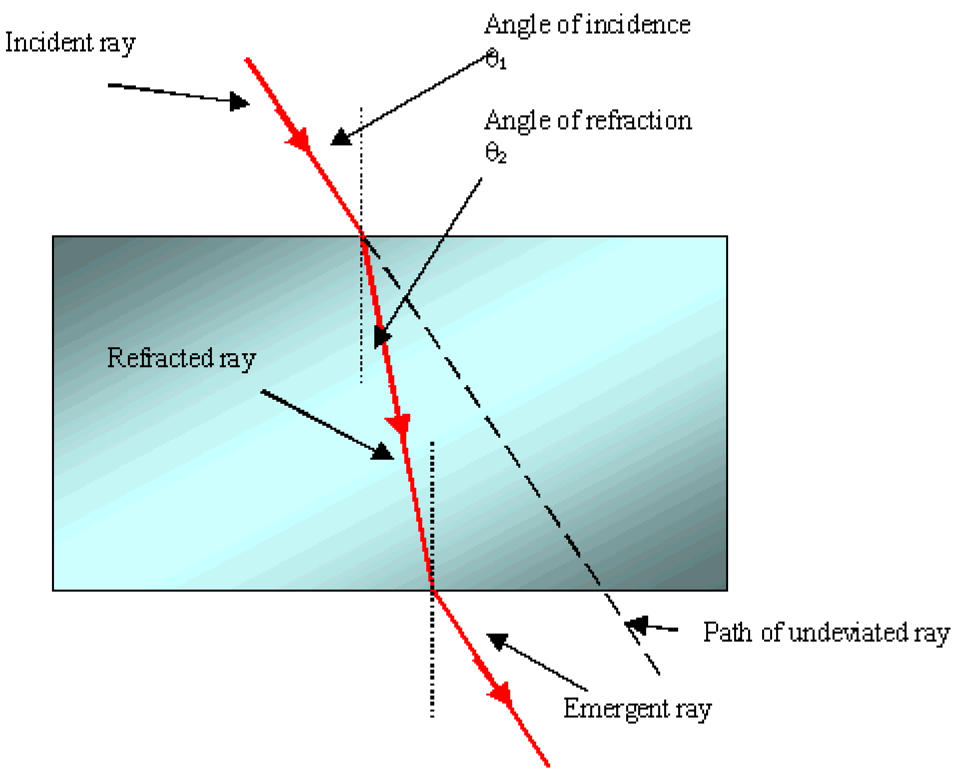


Figure 77 Refraction at an air-glass boundary

We should note the following:

- The ray bends in towards the normal as the ray enters the glass. The angle of incidence is greater than the angle of refraction.
- On leaving the glass the light regains its original speed. The **emergent** ray bends **away** from the normal. In this case the angle of incidence is less than the angle of refraction.
- The path of the emergent ray is parallel to the path of the undeviated ray (where the ray would have gone if there hadn't been a piece of glass in the way). This only happens when the sides of the object are parallel. It wouldn't happen in a prism.
- If the boundary is indistinct, then refraction is gradual with the change occurring over a long distance. Earthquake waves refract in curves due to the gradually changing density of rocks.

- Different wavelengths of light are refracted by different amounts; blue light is refracted the most, red light the least.
- When doing calculations, we generally assume the light is yellow.
- There is always a weak reflected ray.

7.064 Refractive Index

The **refractive index** given the physics code n and has no units. We can also describe the **absolute refractive index** as **the ratio of the speed of light in a vacuum to the speed of light in an optical medium**. Glass has a refractive index of 1.5, so the speed of light is:

$$3.0 \times 10^{8} \text{ m s}^{-1} \div 1.5 = 2.0 \times 10^{8} \text{ m s}^{-1}.$$

Before we carry on, we need to be sure of what these terms mean:

- The **absolute refractive index** is the ratio compared with the refractive index of a vacuum. (n for a vacuum = 1.000;)
- The **relative refractive index** is the ratio of the absolute refractive index of one material compared to that of another, for example from water to glass.

From this we can write:

$$n_1 = \frac{c}{c_1}$$

.... *Equation 41*

and

$$n_2 = \frac{c}{c_2}$$

.... *Equation 42*

We can rearrange each equation (*Equations 41* and *42*) and combine the two to write:

$$n_1 c_1 = c = n_2 c_2$$

.... *Equation 43*

So, we can write:

Speed of light in material 1

Refractive index of material 2

$$\frac{c_1}{c_2} = \frac{n_2}{n_1}$$

Speed of light in material 2

Refractive index of material 1

.... *Equation 44*

Note that:

- Absolute refractive index is the **ratio** between the speed of light in a **vacuum** and speed of light in a **material**.
- It is always **greater than 1.0**, as there is no material optically less dense than a vacuum.
- The **relative refractive index** is the ratio between the speed of light in material 1 and the speed of light in material 2.
- Relative refractive index can be **less** than 1.
- There are **no units** for refractive index as it's m s^{-1} / m s^{-1}.

7.065 Snell's Law

It was the Dutch physicist Willebrord Snel van Royen (1580 – 1626), who in 1621 rediscovered the quantitative relationship between the angle of incidence and the angle of refraction. It was initially worked out by Ibn Sahl (940 – 1000), an Arabic scholar, in 984. If we double the angle of incidence, we do NOT double the angle of refraction.

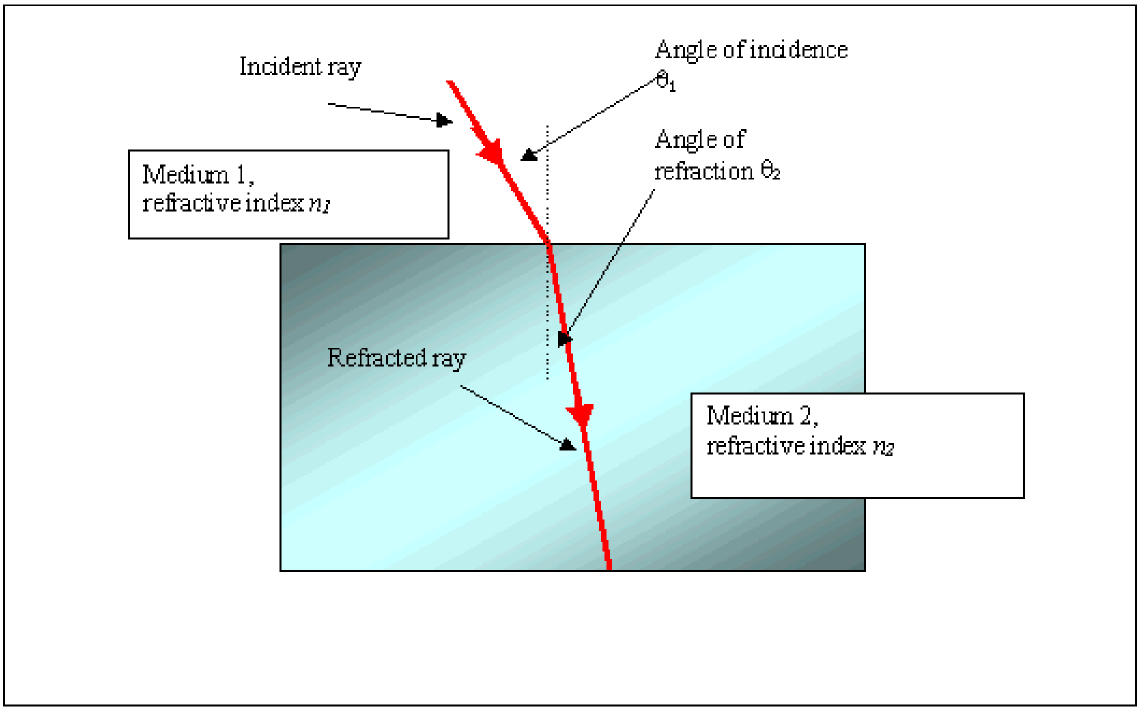


Figure 78 Snell's Law

The ratio n_2/n_1 is the **relative refractive index**, which we can also define as the **ratio of the sines of the angles of incidence and refraction**:

$$\frac{n_2}{n_1} = \frac{c_1}{c_2} = \frac{\sin \theta_1}{\sin \theta_2}$$

.... *Equation 45*

We can write this formula in a more useful form:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$ ………… *Equation 46*

The ratio n_2/n_1 can also be written as ${}_1n_2$. This means the refractive index of light going in from material 1 to 2.

$$ {}_1n_2 = \frac{n_2}{n_1} = \frac{c_1}{c_2} = \frac{\sin\theta_1}{\sin\theta_2} $$

.... *Equation 47*

Remember that each wavelength of light has a slightly different refractive index. Generally, we use **yellow light**.

The table shows some absolute refractive indices for some common materials:

Medium	*Refractive Index*
Vacuum	1.000
Air	1.003
Ice	1.30
Water	1.33
Glass	1.50
Diamond	2.42

The absolute refractive index will always be greater than 1.0 as light cannot travel any faster than it does in a vacuum. It is possible for particles to travel faster than light in a material (but NOT in a vacuum). The result of this will be little flashes, the light equivalent of a sonic boom, called **Cherenkov radiation** (*Figure 79*). This is why radioactive materials appear to make water glow.

Figure 79 Cherenkov radiation (Photo: United States Nuclear Regulatory Commission, Wikimedia Commons)

Now let us have a look at what happens when a ray passes from a dense medium to a less dense medium. Diagram A shows the ray going from air to glass, while Diagram B shows the ray leaving the glass and going into air (*Figure 80*).

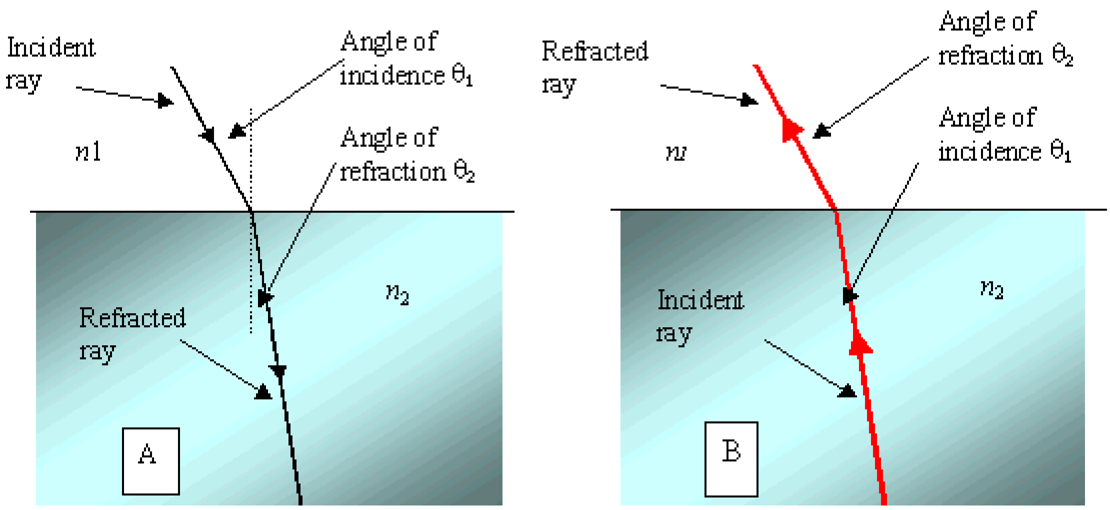


Figure 80 Comparison of light ray passing into an optically more dense medium and a less dense medium

In Diagram A we have established that:

$$ {}_1n_2 = \frac{\sin\theta_1}{\sin\theta_2} $$ .... *Equation 48*

In Diagram B we see:

- Ray of light travels faster in the optically less dense medium
- The ray bends away from the normal.
- The angle of incidence is **less** than the angle of refraction.

The relative refractive index going from medium 2 to medium 1 is ${}_2n_1$, so we can write:

$$ {}_2n_1 = \frac{\sin\theta_2}{\sin\theta_1} $$ .... *Equation 49*

Therefore:

$$ {}_1n_2 = 1 \div {}_2n_1 $$ .... *Equation 50*

For example, the refractive index from air to water is 1.33, while the refractive index from water to air = 1/1.33 = 0.75

7.066 Critical Angle

We have seen how a ray of light passing from glass to air bends away from the normal. If we increase the angle of incidence, the angle of refraction increases more (Diagrams A and B, *Figures 81* and *82*):

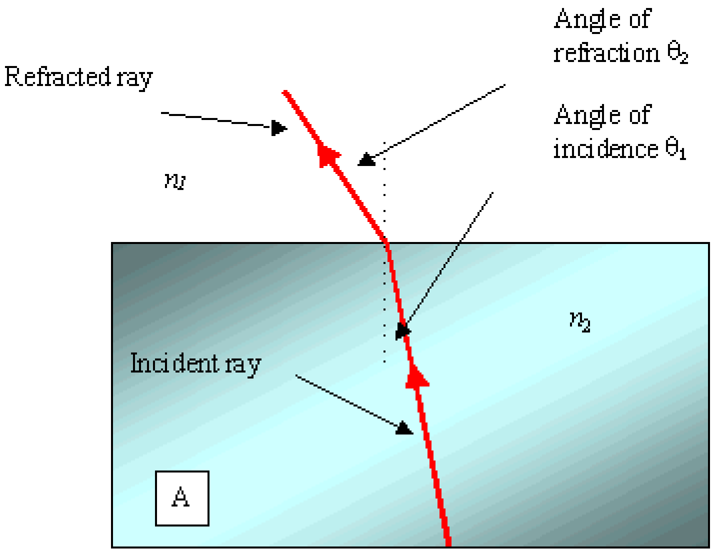


Figure 81 Light ray passing from glass to air

Angle of refraction getting bigger:

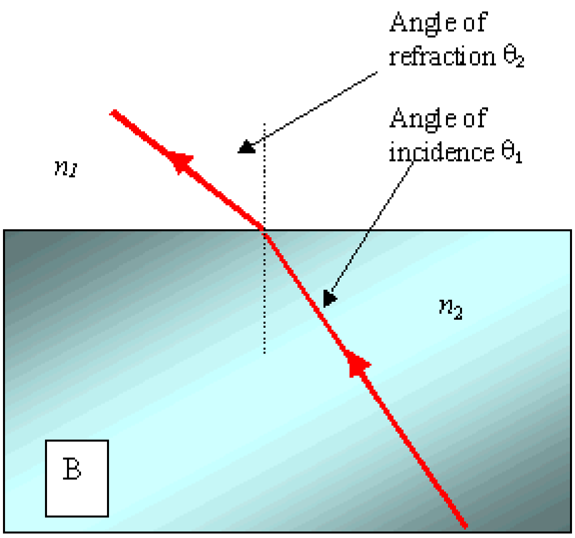


Figure 82 Increasing the angle of incidence of the rat going from glass to air

At a particular value of angle of incidence, the angle of refraction is 90°, as shown in Diagram C (*Figure 83*). This particular angle of incidence is called the **critical angle**.

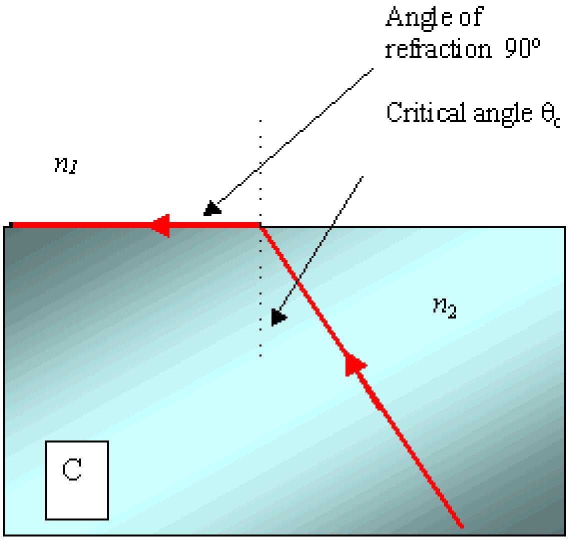


Figure 83 Critical angle

Above the critical angle we get **total internal reflection**. There is no transmission of light at all. See diagram D (*Figure 84*).

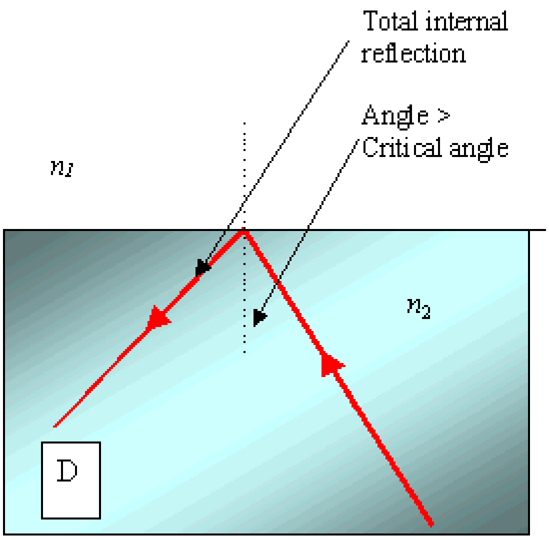


Figure 84 Total internal reflection

Above the critical angle we get **total internal reflection**.

When there is total internal reflection, there is no transmission of light at all. We know that:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \text{………… Equation 51}$$

At the critical angle:

$$n_2 \sin \theta_c = n_1 \sin 90 \quad \text{……….. Equation 52}$$

Since $\sin 90 = 1$, we write:

$$n_2 \sin \theta_c = n_1 \quad \text{…………. Equation 53}$$

Therefore

$$\sin \theta_c = \frac{n_1}{n_2}$$

.... *Equation 54*

We can write this as:

$$\sin \theta_c = \frac{1}{n_2/n_1}$$

.... *Equation 55*

Since n_2/n_1 is the relative refractive index ${}_1n_2$, we can write:

$$\sin \theta_c = \frac{1}{{}_1n_2}$$

.... *Equation 56*

Or we can write:

$$\theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right)$$

.... *Equation 57*

If you struggle with this, the best way to tackle it is to use:

$$n_2 \sin \theta_c = n_1 \sin 90 \quad \text{………… Equation 58}$$

Note:

- Make sure that the bigger refractive index goes downstairs in the equation, otherwise you will get a sine value greater than 1. This will not work.
- So, if you get $\sin^{-1}$ (1.06), for example, you have done it wrong.

7.067 Total Internal Reflection in Optical Fibres

An example of the use of total internal reflection is in **optical fibres**. At their simplest, optical fibres are long thin strands of glass that carry light from one end a long distance to the other (*Figure 85*). The light can be guided round corners using total internal reflection.

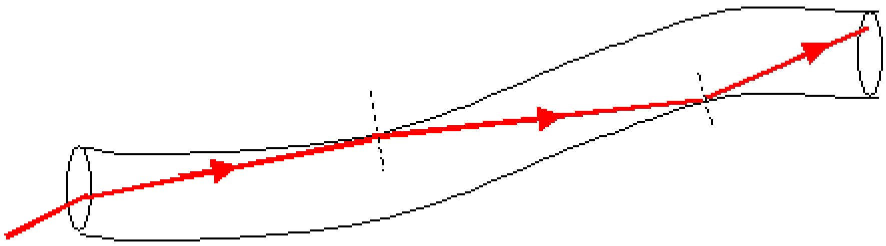


Figure 85 An optical fibre

There are two problems:

- Ordinary glass is not very good. It is impure and has a **high attenuation** coefficient, losing most of its intensity after a short distance. Red and blue light is absorbed, leaving only green.
- Wide fibres tend to **smear** the signals as there is a path difference between the rays that go down the middle of the fibre and those that bounce from side to side.

The problems can be overcome by:

- Using very high purity glass.
- Using **infra-red** transmitters.
- The light is channelled along a very thin central fibre that is clad with glass of a lower refractive index. Fibres with a **step** index have one layer of cladding, while **graded index** fibres have several layers of cladding. The best fibres are called **monomode** fibres, with a channel of no more than 1.25 mm, which is very narrow. The path difference between the axial ray and the reflected rays is negligible.

Optical fibres are amazingly flexible and strong. They mounted in a polythene tube for further strength. Optical fibres offer many advantages over wires (*Figure 86*).

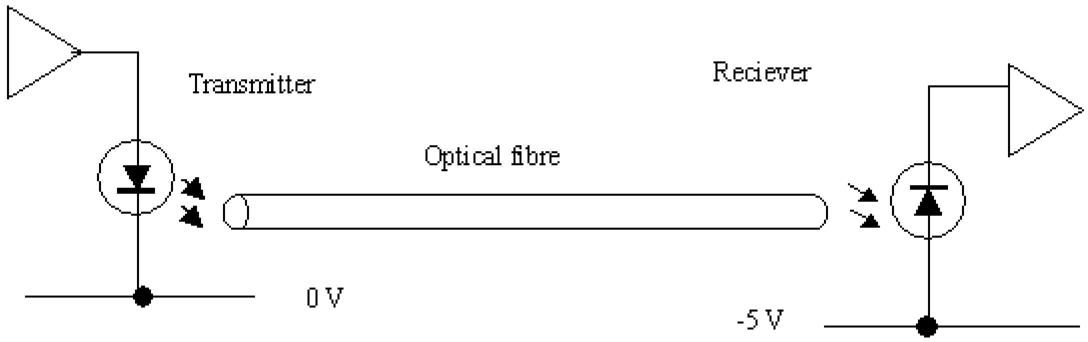


Figure 86 Simple optical fibre data transmission system

Fly by Light

If we use a copper wire to transmit data from several sensors, we have to be very careful to **screen** it from unwanted signals. Ordinary wires are very prone to this, especially in long runs. Screening is easy enough; we simply surround the wire with an **insulating** layer and then **copper braiding** to **shield** the wire (*Figure 87*). We have a **coaxial** cable.

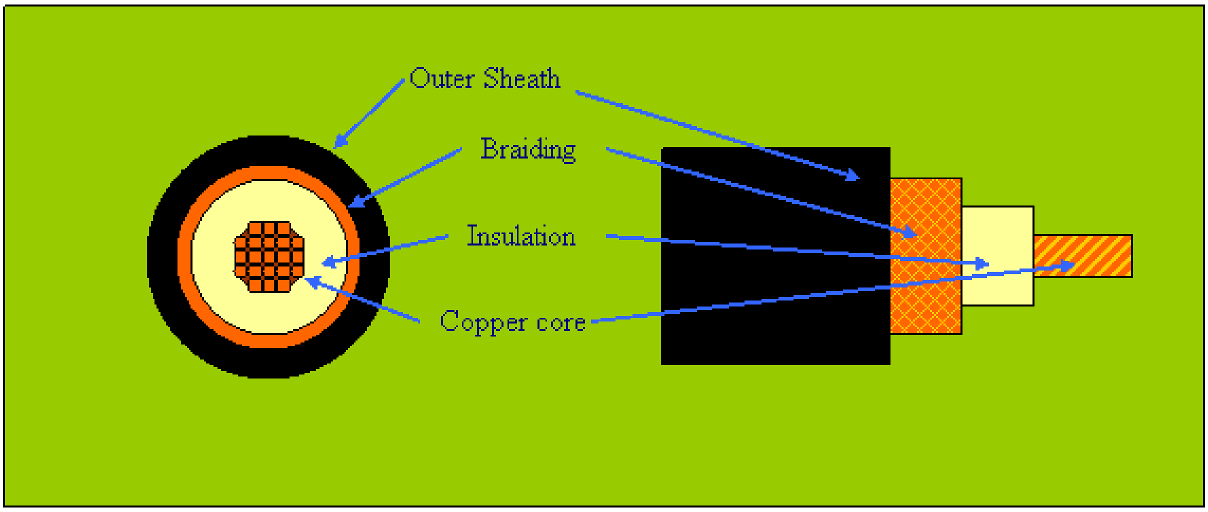


Figure 87 Coaxial copper cable

The problem with coaxial cable is that:

- It is heavy.
- It is bulky. Lots of coaxial cables take up a lot of space.

More recently there have been projects that use **fly-by-light technology**. This uses **optical fibres**, which are much less heavy and take up a lot less space; several optical fibres can take the space of one coaxial cable. An optical fibre looks like this (*Figure 88*).

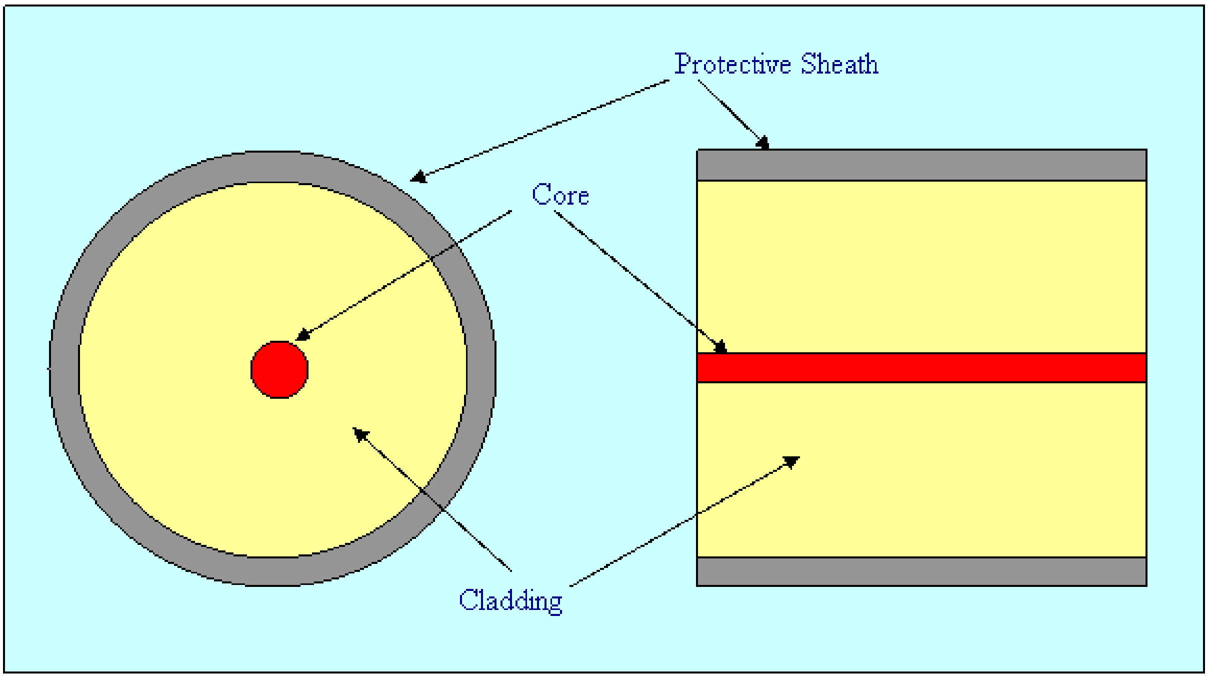


Figure 88 An actual optical fibre

The fibres are 50 μm across (including the sheath). The core has a diameter of 5 μm (5×10^{-6} m).

7.068 Optical Fibres

Optical fibres work by **total internal reflection**. The light ray makes a certain angle of incidence when it hits the boundary of an optically dense material (like glass) and an optically less dense material (like air). If this angle is greater than the critical angle, the ray is totally internally reflected. The critical angle, θ_c, is determined by the formula:

$$_1n_2 = \frac{1}{\sin \theta_c}$$

.... *Equation 59*

Where $_1n_2$ is the physics code for the refractive index going from material 1 to material 2.

We can rewrite the equation in a more user-friendly way:

$$\theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right)$$ .... *Equation 60*

Or we could go back to our old friend:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$ ………….. *Equation 61*

The rays of light should travel like this (*Figure 89*).

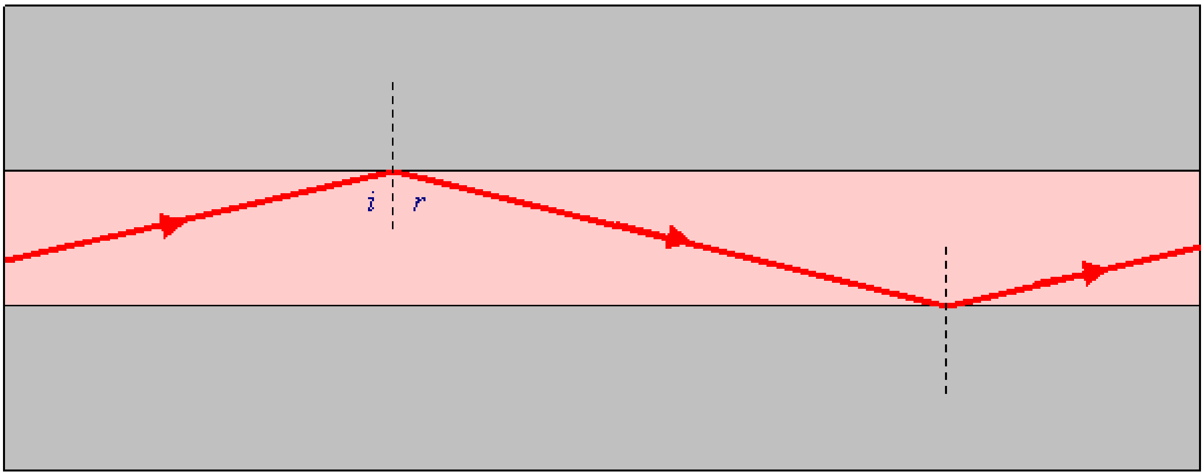


Figure 89 Light rays should travel like this

But instead, light rays can travel several paths (*Figure 90*).

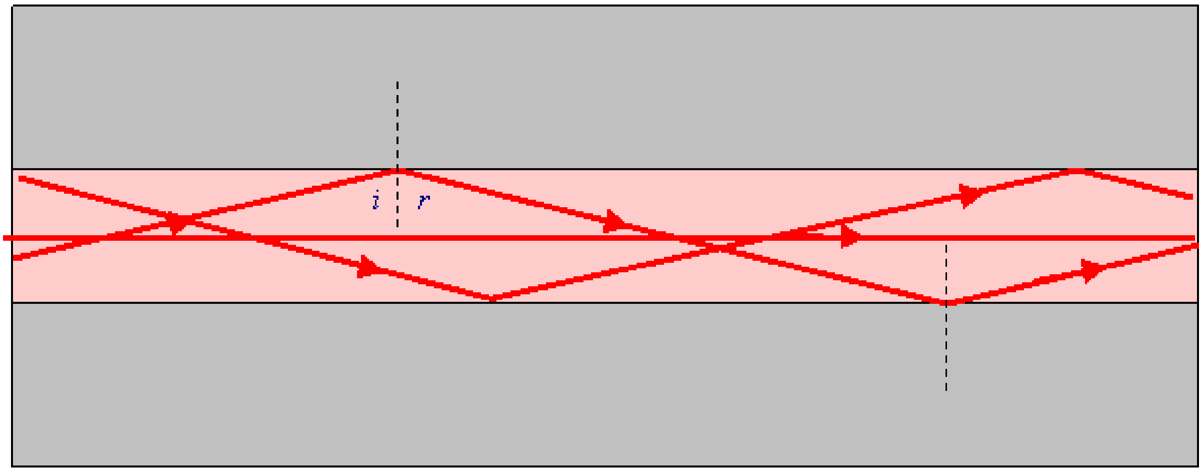


Figure 90 Light rays travelling down an optical fibre with different paths

This means that the light rays can arrive at different times, resulting in **dispersion** or **smearing**. The signal that was sharp when it left the transmitter is smeared. It is called modal dispersion (*Figure 91*).

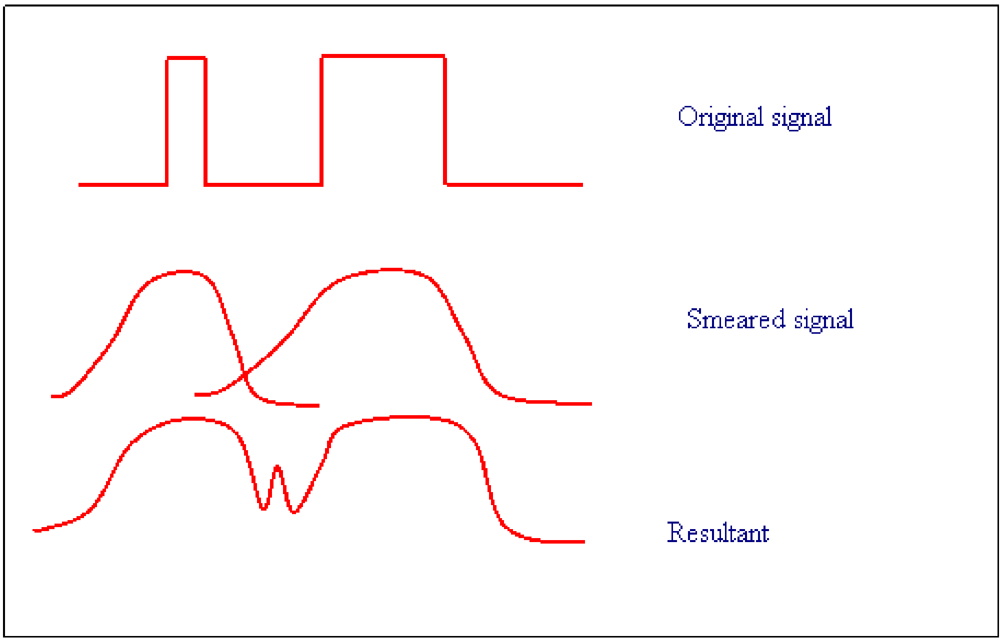


Figure 91 Smearing

The picture shows us how the signal can be unacceptably distorted and even produce spurious signals that were not there. The problem can be resolved by **cladding** the core with a material of slightly lower refractive index (*Figure 92*). For example, the core might have a refractive index of 1.6, while the cladding has a refractive index of 1.4. This is called a **step index** fibre.

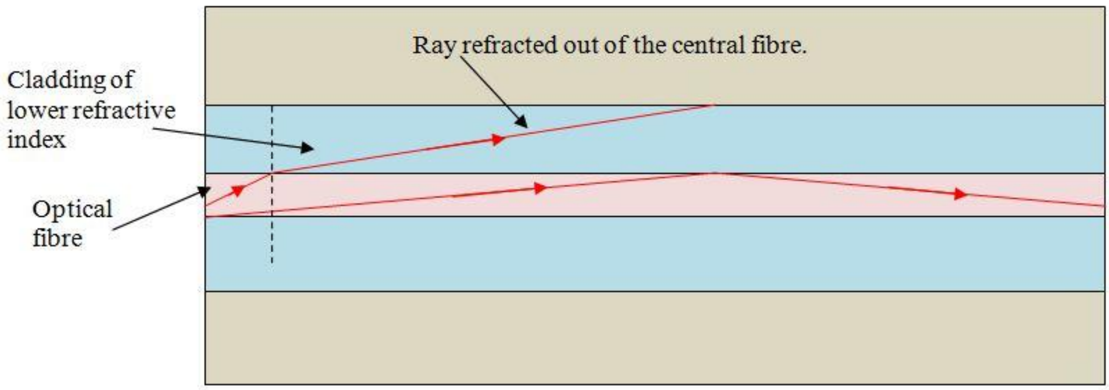


Figure 92 Step index fibre

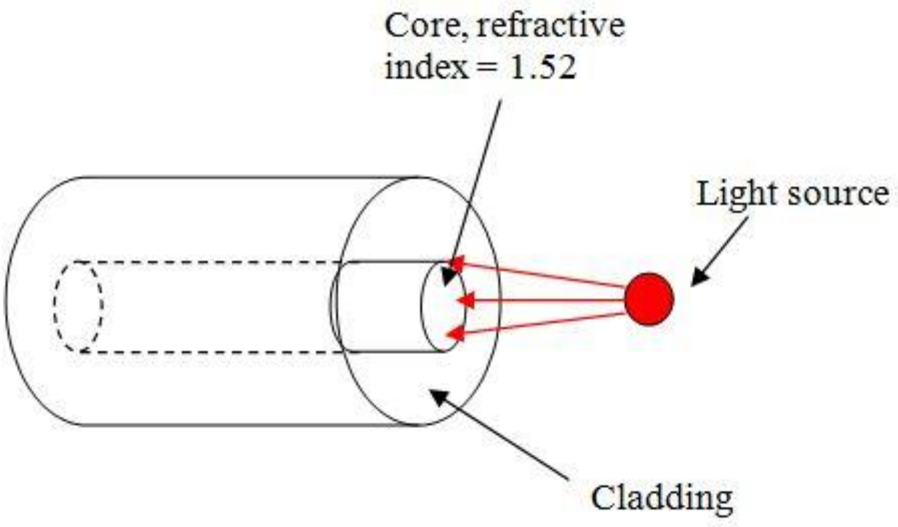
Dispersion can be reduced further by use of a **graded index** or **multimode** fibre. Some light is passes down the middle, which has a higher refractive index, therefore slower rate of travel. With clever manipulation of the refractive indices, the ray travelling down the middle can be made to arrive at the same time as the ray that goes from side to side. They can meet with a time difference of about 1 ns km^{-1}. In aircraft where the distances are generally less than 50 metres, this is not too bad.

Monomode fibres are designed such that the rays pass only down the middle. If the light were perfectly **monochromatic**, i.e. of one wavelength only, the rays would all arrive at the same time. However, even the best lasers produce a slight spread, and since refractive index varies with wavelength, there can be slight differences in arrival times, leading to smearing. This is called **chromatic dispersion**.

Material dispersion happens when different wavelengths interact with the material of the optical fibre in slightly different ways. This can lead to smearing of the signal.

Worked Example

The picture shows light entering into a straight length of step-index optical fibre.



The critical angle between the core, refractive index 1.52, and the cladding is 59°.

(a) Calculate the refractive index of the cladding.

(b) What happens to a light ray that strikes the core/cladding boundary at an angle of less than 59°?

(c) What happens to a light ray that strikes the core/cladding boundary at an angle of greater than 59°?

Answer

(a) Use our old friend:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$n_1 \sin 90 = 1.52 \sin 59$$

$$n_1 = 1.52 \times 0.857 = \mathbf{\underline{1.30}}$$

(b) The light ray will be refracted into the outer cladding.

(c) The light ray will be totally internally reflected.

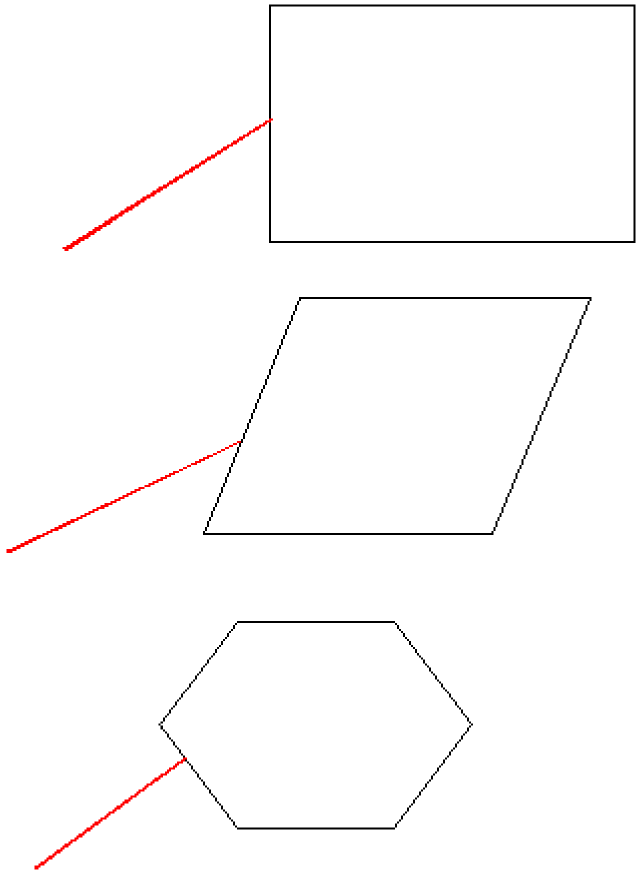
Although we have looked at light, all these phenomena can be observed with other forms of waves, e.g.

- radio waves.
- water waves.
- sound waves.

Tutorial 7.06 Questions

7.06.1

Can you complete these diagrams showing the refracted ray and the emergent ray?



7.06.2

What is the speed of light in glass, refractive index 1.5, when the speed of light in air is 3.0×10^8 m s^{-1}?

7.06.3

What does ${}_2n_1$ mean?

7.06.4

A ray of light strikes an air-glass boundary at an incident angle of 30°. If the refractive index of glass is 1.5, what is the angle of refraction?

7.06.5

What is the relative refractive index when a light ray passes from glass (RI = 1.5) to air?

7.06.6

What is the angle of refraction and what is the angle through which the ray is deviated when light passes at an incident angle of 48° into water of refractive index 1.33 from air at a refractive index of 1.00?

7.06.7

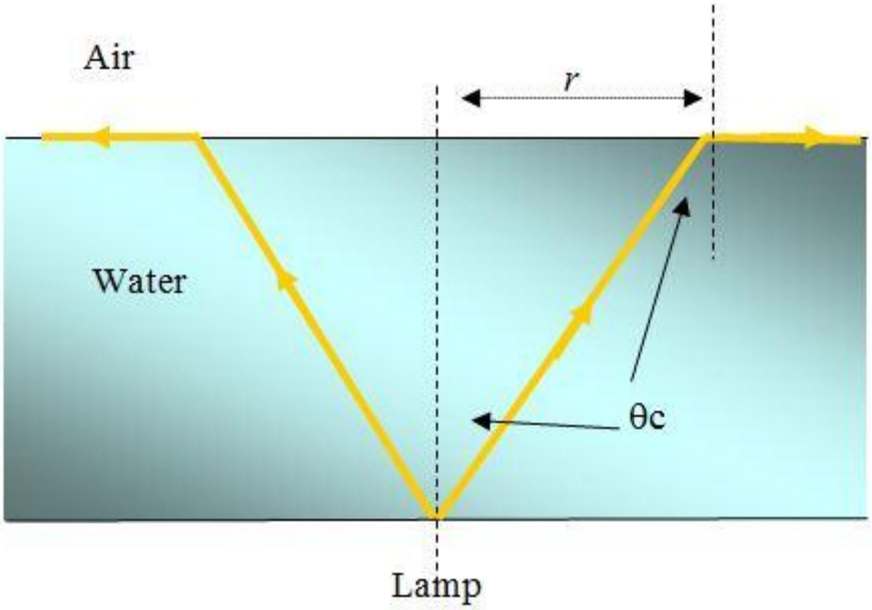
What is the critical angle of water, of which the refractive index is 1.33?

7.06.8

Rays from a point source of light at the bottom of a swimming pool 1.8 m deep strike the water surface and only emerge through a circle of radius r, as shown in the diagram. If the refractive index between water and air is 1.33, calculate:

(a) the critical angle, θc.

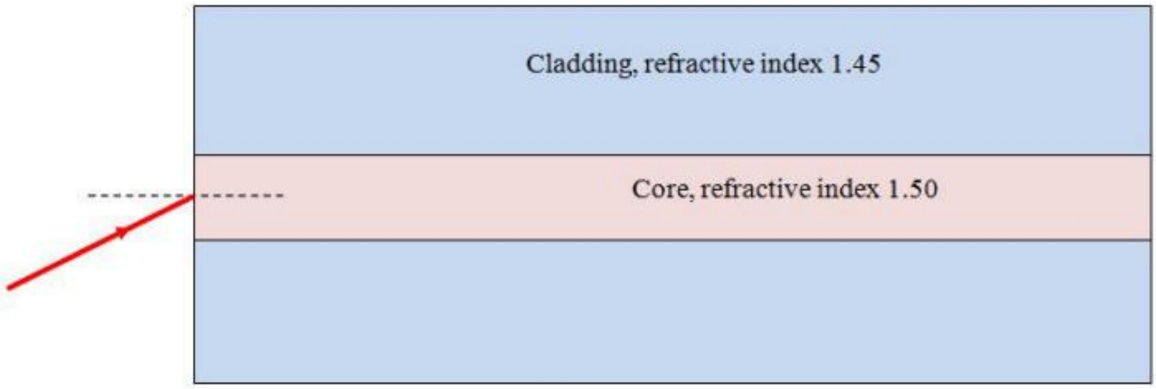
(b) the value of r.



Give your answers to an appropriate number of significant figures.

7.06.9

A stepped index optical fibre has a central core of refractive index 1.50, and a cladding of refractive index 1.45.



A single fine beam of monochromatic light enters the core at an incident angle of 30°.

(a) Calculate the angle of refraction of the light ray as it passes into the fibre.

(b) Calculate the critical angle between the core and the cladding.

(c) Explain whether or not the light will continue to go down the core.

Tutorial 7.07 Interference

All Syllabi

Contents

7.071 Interference of Waves

When two progressive waves of the same type coincide, they **superpose**. The results of the superposition is **interference** (*Figure 93*).

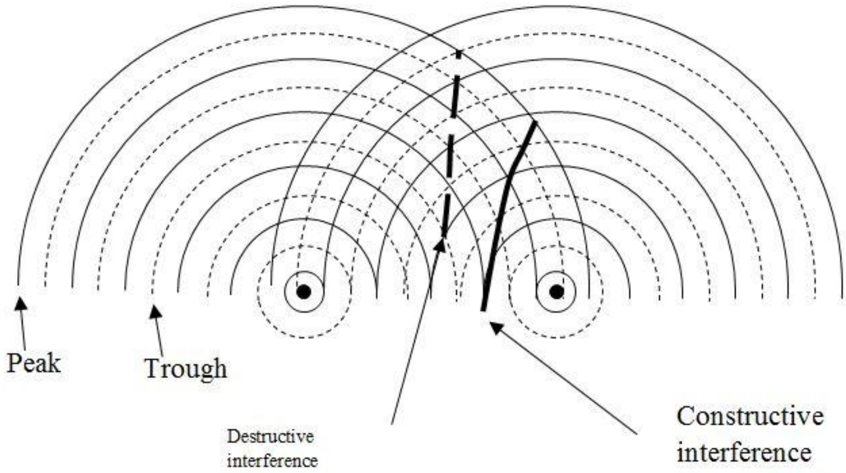


Figure 93 Interference of circular wavefronts from two point sources

We can demonstrate this with water waves in a ripple tank. The two dippers act as two different sources which are in phase and having identical wavelengths and frequencies. We say that the sources are **coherent**. Coherent waves have:

- The same wavelength.
- The same velocity.
- The same frequency.
- The same phase relationship.

The waves are NOT necessarily in phase. The phase relationship is constant.

In the pattern we can see:

- Regions of **constructive interference** where crests meet crests and troughs meet troughs. The amplitude is larger.
- Where the waves are in antiphase, there is **cancellation**.

We should note the following:

- At any point where there is constructive interference, the water is still rising and falling at the same frequency, but with greater amplitude.
- The position of the pattern remains steady, not altering with time.
- The total energy of the system remains constant. While there is no energy in the regions of cancellation, there is more energy in the regions of reinforcement.

We can demonstrate the same thing with sound waves. We set up two loudspeakers driven by the same signal generator. If we use a microphone connected to a CRO, we can detect regions of reinforcement and cancellation. (In reality we don't get complete cancellation.)

We can explain our observations in terms of **path differences** (*Figure 94*). Suppose we go along the centre line between the two sources. At all points we are the same distance from either of the sources. There is zero **path difference**. Since the waves are in phase and produced at the same frequency and travelling at the same speed, they must still be in phase. So, they must **reinforce.**

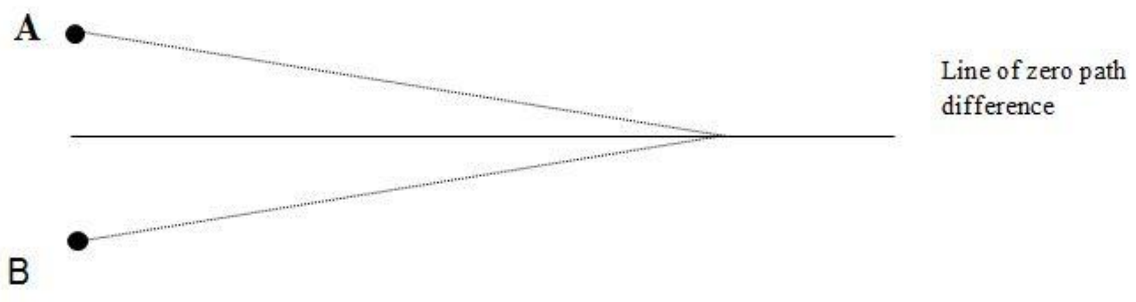


Figure 94 Zero path difference results in reinforcement

We also see regions of constructive interference symmetrically on either side of the centre line. Thus, the waves must be in phase. This is because the waves have a path difference of one or more whole wavelengths. We often describe this in terms of **half wavelengths**, so for there to be constructive interference, there must be a path difference of an **even** number of half wavelengths.

The reverse side of the argument applies to **odd** numbers of half wavelengths. If the path difference is ½ a wavelength or 1 ½ and so on, we get regions of cancellation. This is because the waves are in **antiphase**.

We can demonstrate similar effects with microwaves and sound. In general, if the separation of the sources is smaller compared to the wavelength, the pattern of constructive and destructive interference is more spread out.

The uses of this are not confined to the laboratory. Freak waves in storms can occur due to this. Attempts at **sound deadening** using high speed computers to produce sound waves in antiphase have been successful. These are now used by pilots in high quality headphones used in the noisy environment of an aeroplane cockpit.

7.072 Interference of Light

Getting two **coherent** light sources is extremely difficult due to the nature of the production of light. Light is produced by the excitation of individual groups of atoms in bursts lasting less than nanoseconds ($<1 \times 10^{-9}$ s). These are random so that there is no constancy in the phase relationships, even from a small region of the light source. Although we need not go into the explanation for this, it has been found that the coherence length for two rays of light rarely exceeds 1 mm.

Thomas Young first demonstrated interference in 1801. He didn't have a laser, just a candle. He split the light from a single source into two. In this way he got coherent beams. Using modern laboratory equipment, we can reproduce his set up. A coloured filter prevents dispersion of the light into a spectrum (*Figure 95*).

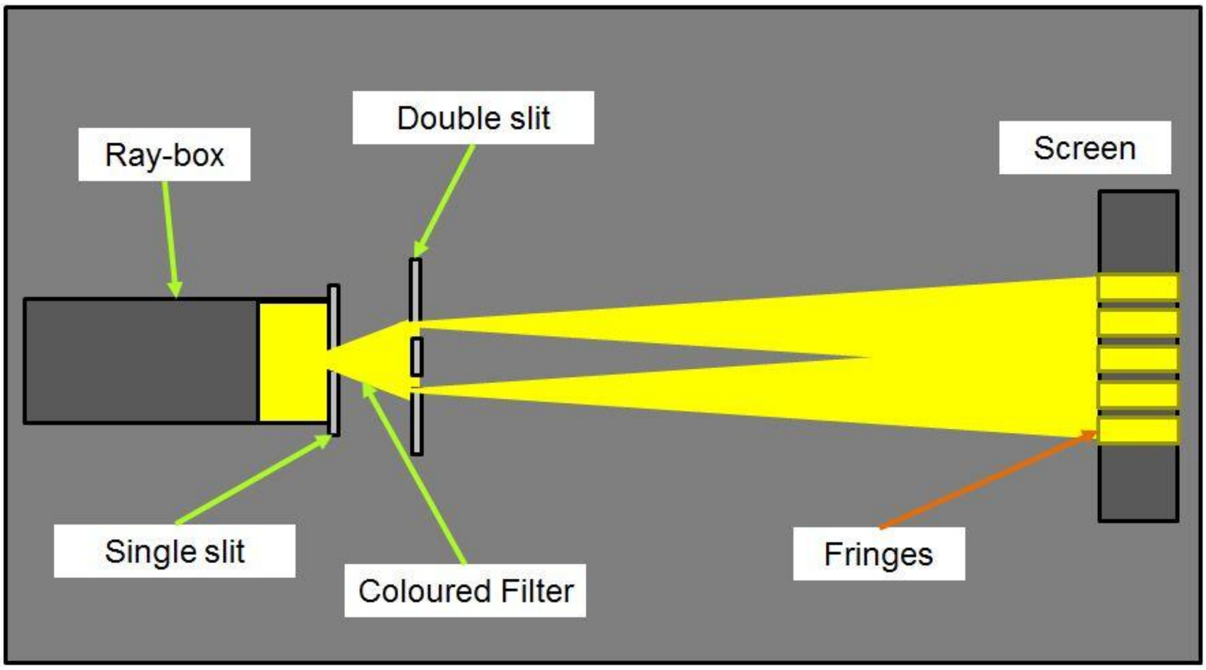


Figure 95 Young's double slit with a ray box

Even with a bright ray-lamp, the results are not exactly convincing. How he managed with a candle…

We have lasers now which make it easy to demonstrate.

The **LASER** produces a single high intensity **monochromatic** (one wavelength) beam where all the waves are in a constant phase relationship. If we can split this, we can easily demonstrate interference effects (*Figure 96*). You are not expected to know how the LASER works, but a detailed discussion can be found in Topic 1.

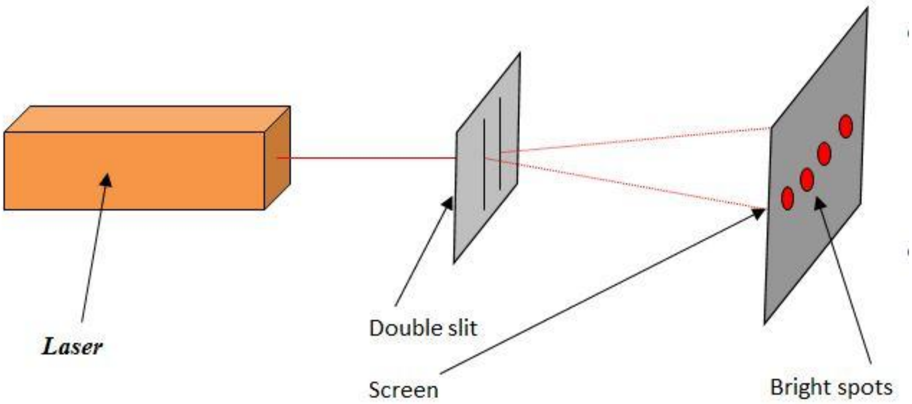


Figure 96 Young's double slits with a laser

When the laser shines on the double slit, all the waves are in phase, so the slits act as coherent sources. In the diagram the point O is the centre point on the screen and is equidistant from the two sources. Therefore, there must be reinforcement, because the waves arrive in phase. A **bright fringe** is produced. This fringe is made by waves whose path difference is zero (*Figure 97*).

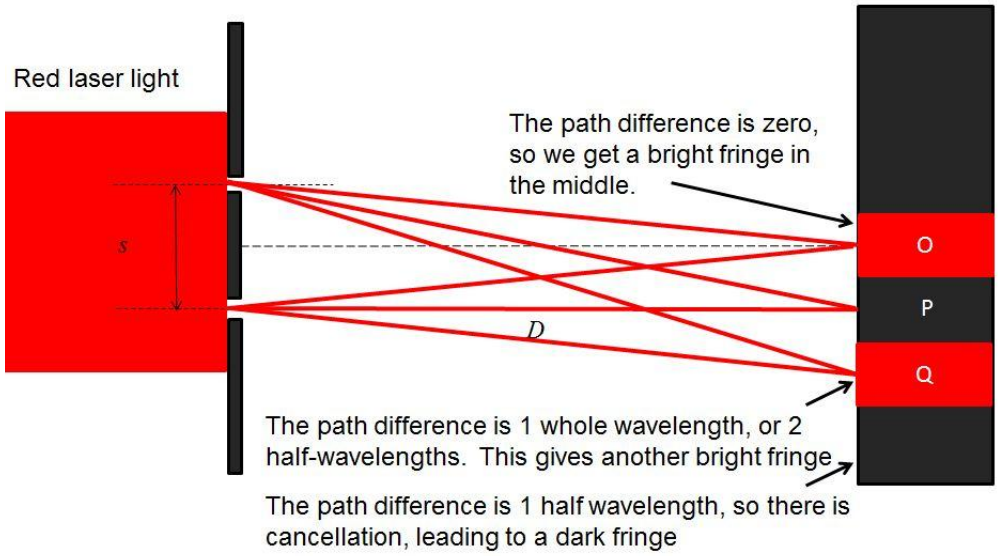


Figure 97 Fringes from Young's double slits

At **P** there is a **dark** fringe, where there is no light. The waves must be in antiphase to cancel out so the path difference must be one half wavelength. At **Q**, the path difference is two half wavelengths, so another bright fringe or **maximum** is found. Where the path difference is an odd number of half wavelengths, **minima** are found; even numbers of half wavelengths produce **maxima**.

It was found that the wavelength could be found according to the formula:

$$\lambda = \frac{ws}{D}$$

.... *Equation 62*

- λ is the wavelength (m)
- w is the **fringe spacing** (m)
- s is the **slit spacing** (m)
- D is the distance from the slits to the screen (m).

Worked example

In a Young's Double Slit experiment using a laser of wavelength 638 nm, the screen is placed at a distance of 2.5 m from the double slit. If the slit separation is 0.50 mm, what is the distance between fringes?

Answer

Formula first:

$$\lambda = \frac{ws}{D}$$

Rearranging:

$$w = \frac{D\lambda}{s}$$

l = 638×10^{-9} m; $s = 0.50 \times 10^{-3}$ m; $D = 2.50$ m

$$\Rightarrow w = \frac{2.50 \text{ m} \times 638 \times 10^{-9} \text{ m}}{0.50 \times 10^{-3} \text{ m}} = 0.00319 \text{ m} = \mathbf{3.2\ mm}$$

Remember always to convert nanometres to metres.

1 nm = **1×10^{-9} m**. You will avoid this bear-trap.

Young's Double Slits is a required practical for the first year of the A-level course. You will use a laser for this experiment.

Tutorial 7.07 Questions

7.07.1

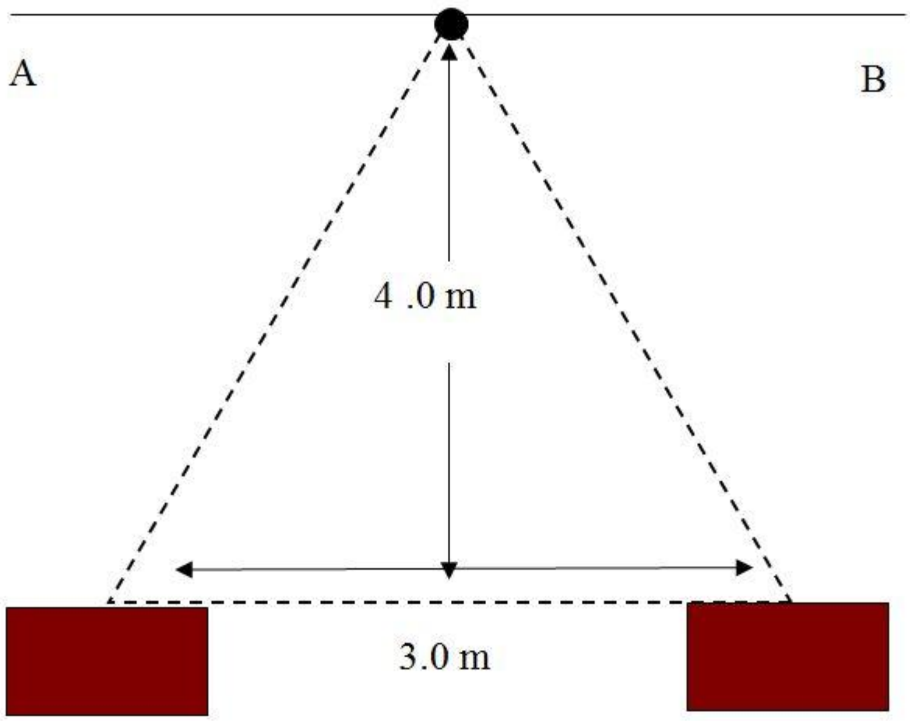
What is meant by the term path difference? How is it measured?

7.07.2

What path differences are needed for constructive and destructive interference? Explain your answers.

7.07.3

Two loudspeakers are set 3.0 m apart in a room. A microphone connected to a CRO is placed at the apex of a triangle 4 m from the line separating the two loudspeakers as shown below:



The microphone picks up sound waves of a very large amplitude. If the microphone is moved along the line AB by 10 cm to the right of the central point, a point of minimum amplitude is found. What is the wavelength of the waves? What is the frequency of the signal? Speed of sound is 340 m s^{-1}. (*Hint* - you will need to do some geometry!)

7.07.4

What is meant by coherent wave sources?

7.07.5

In a Young's Slit experiment, laser light of 630 nm is used to illuminate two slits of separation 0.20 mm. Calculate the fringe separation on a screen 3 m away.

Tutorial 7.08 Diffraction	
All Syllabi	
Contents	
7.081 Diffraction Effects	7.082 Single Slit Diffraction
7.083 Central Bright Fringe	7.084 Resolution
7.085 The Diffraction Grating	7.086 Diffraction Gratings and Spectra
7.087 Spectroscope	7.088 Resolvance (IB Syllabus)

7.081 Diffraction Effects

If we pass waves through a single slit, we observe that the waves spread out due to **diffraction** (*Figure 98*)

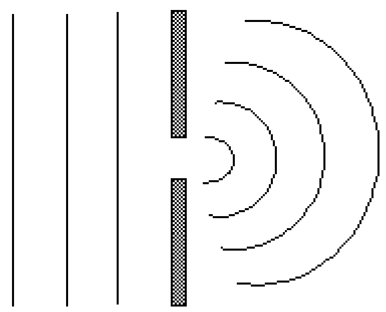


Figure 98 Diffraction

Notice:

- If the slit is narrow, the diffraction is more marked.
- The wavelength remains the same.
- Diffraction does not need a slit. Waves can bend round a barrier by diffraction. Radio signals can be picked up behind hills for this reason.
- The longer the wavelength, the more the waves will diffract.
- All waves diffract.

7.082 Single Slit Diffraction

We can show the diffraction of light due to a **single slit**. (We must be careful not to confuse this with Young's double slits.) If we have a wide slit, we see just a single bright region with sharp edged shadows (*Figure 99*).

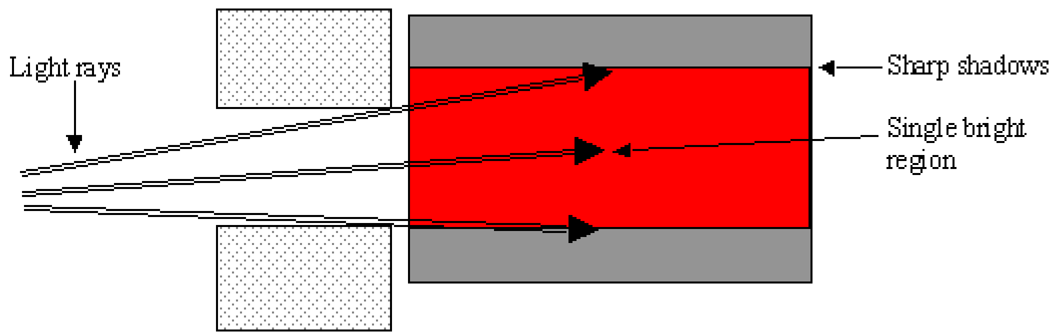


Figure 99 Single slit diffraction through a wide slit

If we make the slit narrower, we see a pattern emerging with a bright central region, and alternating light and dark bands. The narrower the slit, the marked the effect. The central bright region becomes dimmer as well because less light is transmitted (*Figure 100*).

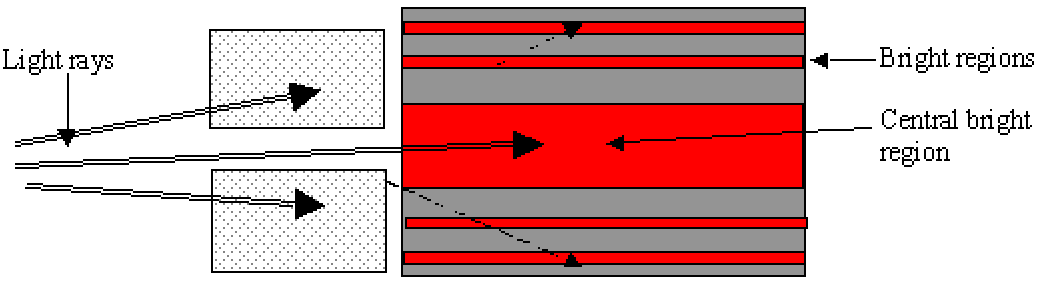


Figure 100 Diffraction through a narrow single slit

If the light is **monochromatic**, the bands will be of the same colour. Red light has a broader pattern than blue light, suggesting that the diffraction effect increases with wave length. If we use white light, the central band is white, with the fringes being overlapped with the spectrum of colours. This is called **Frauenhoffer diffraction.**

We can plot a graph to show the intensity, and we see a bright central maximum, with subsidiary maxima either side. We can explain the effect of diffraction using the idea of **secondary wavelets**. In the middle these form a plane wavefront. At the edges, circular wavefronts move into the shadow region. The maxima and minima are caused respectively by constructive and destructive interference (*Figure 101*).

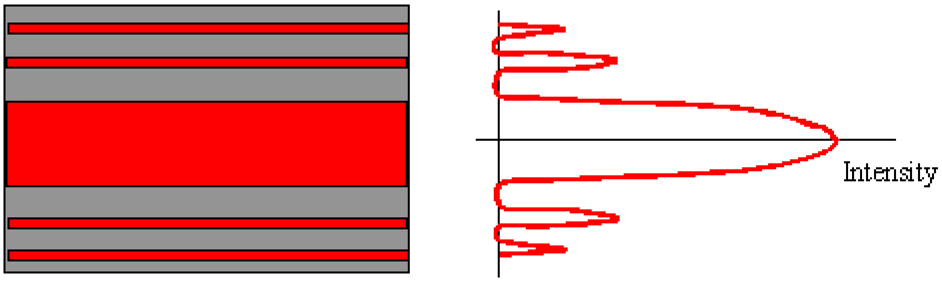


Figure 101 Intensity of diffracted light through a single narrow slit

We can work out the angle of diffraction using a simple equation:

$$\sin\theta = \frac{\lambda}{a}$$

.... *Equation 63*

where θ is the angle, λ is the wavelength and a is the width of the aperture.

Read the question to see if it's single or double slit. The key thing is that the pattern from **double slits** is due to **interference**. From a **single slit**, it's due to **diffraction**.

Single slit diffraction has NOTHING to do with Young's Double Slits.

Worked Example

A piano note of 256 Hz is played. It is heard through a door 200 cm wide. What is the maximum angle of diffraction that will occur if the speed of sound in air is 336 m/s?

Answer

We need to know the wavelength at first:

$$\lambda = c/f \Rightarrow \lambda = \frac{336\ \text{m s}^{-1}}{256\ \text{Hz}} = 1.31\ \text{m}$$

Now we can work out θ:

$$\sin q = \frac{\lambda}{a} = \frac{1.31\ \text{m}}{2.0\ \text{m}} \Rightarrow \sin\theta = 0.655 \Rightarrow \theta = \mathbf{41^\circ}$$

If the door were less than 1.31 m wide, diffraction could not occur because $\sin\theta$ would be greater than 1, which is impossible.

7.083 Central Bright Fringe

The width of the central bright fringe in single slit diffraction is shown in the diagram below (*Figure 102*).

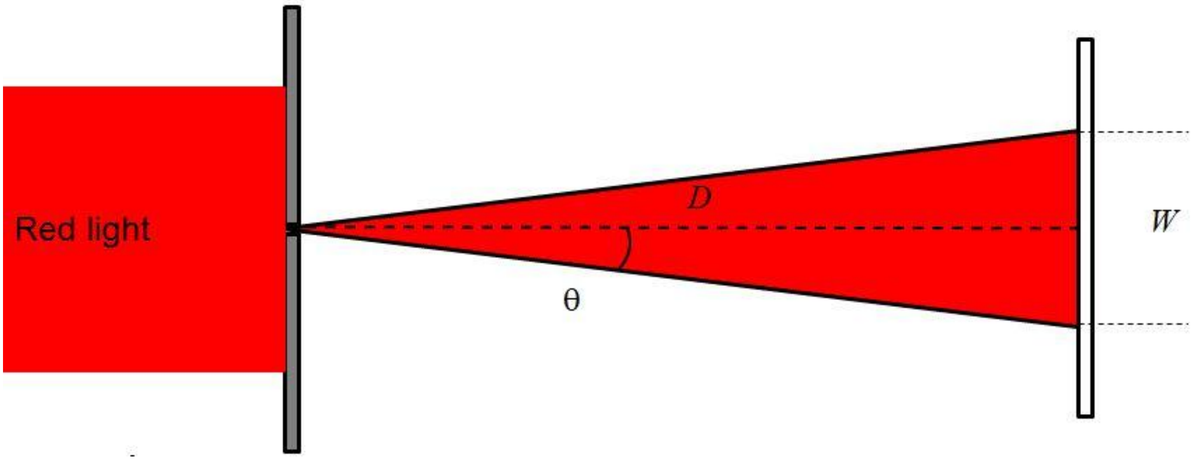


Figure 102 Central bright fringe

From the diagram above, we can see:

$$D \tan\theta = \frac{W}{2}$$

.... *Equation 64*

For very small angles in **radians**, tan θ= sin θ. Since:

$$\sin\theta = \frac{\lambda}{a}$$

.... *Equation 65*

we can combine *Equations 64* and *65* and write:

$$D\frac{\lambda}{a} = \frac{W}{2}$$

.... *Equation 66*

Rearranging *Equation 66* gives us:

$$W = 2\frac{D\lambda}{a}$$

.... *Equation 67*

If white light is used, the pattern is rather messy (*Figure 103*).

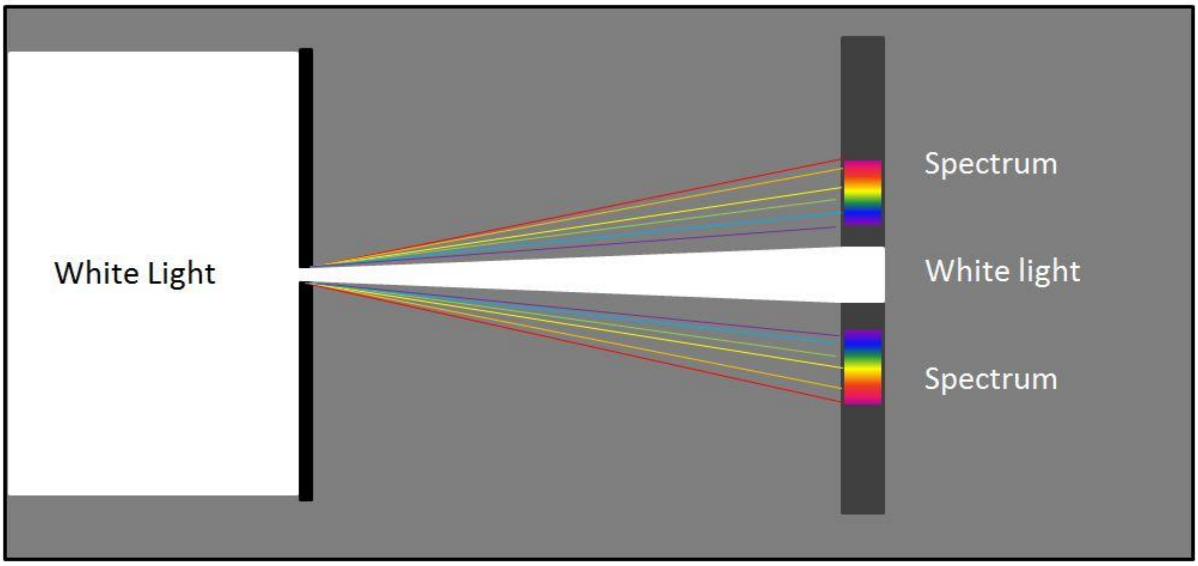


Figure 103 Using white light, the diffraction pattern is messy

The central bright spot is white. Then the next fringes show the **spectra** of visible light. Red light diffracts the most, and violet light the least. This diagram shows the first bright fringes only.

7.084 Resolution (*Extension*)

The single slit diffraction equation can be used, with modification, to determine the limit to which an optical instrument can resolve. This is called the **resolving power**. (This is sometimes known as **Rayleigh's Criterion**.) For a light microscope, the theoretical limit is about 1 mm, so the microscope cannot be used to view atoms. A beam of electrons is regarded as having wave properties. So, an electron microscope has a much bigger resolution, as the wavelength of the electron beam is much shorter.

Consider two objects very close together (*Figure 104*):

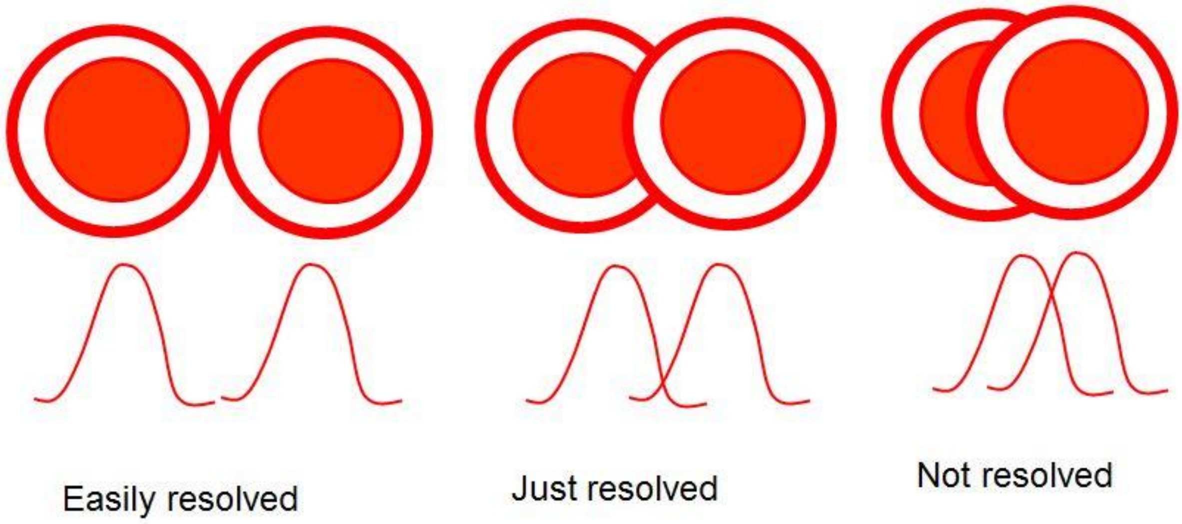


Figure 104 Resolving objects very close together

The red spot in the middle and the intensity peaks below show the central bright region. If the central bright regions are separate, they can be easily resolved. If they touch, they can be just resolved. If they overlap, they cannot be resolved.

Radio waves diffract round hills, which is why we can pick up radio signals behind hills, even though there is no direct line of sight between the transmitter and the receiver.

The angular separation at which the two objects are resolved is given by the formula:

$$\theta = \frac{\lambda}{D}$$

.... *Equation 68*

[θ - angular separation (rad); λ - wavelength (m); D - aperture width (m)]

The angle term θ must be in radians. This is because $\sin\theta \approx \theta$ in radians. If the angle is in **degrees**, the relationship becomes:

$$\sin\theta = \frac{\lambda}{D}$$

.... Equation 69

In astronomy, where:

$$\theta < \frac{\lambda}{D}$$

.... Equation 70

the two stars cannot be resolved. To improve the resolution of a telescope, we need to have a large aperture and a short wavelength.

In the AQA syllabus, the aperture is regarded as a **single slit**. In other syllabuses, the aperture is regarded as a **circular disc**. In which case, the relationship is modified to:

$$\theta = 1.22\frac{\lambda}{D}$$

.... Equation 71

In practice, although telescopes have much better resolution than the eye, this is limited by the atmosphere. Telescopes have large apertures to allow as much light to get in as possible.

Derivation of the Equation

We know that the diffraction equation for a diffraction grating is:

$$n\lambda = d \sin\theta$$

.... *Equation 72*

Where:

- n = number of orders
- λ = wavelength (m)
- d = slit spacing (m)
- θ = angle (rad)

In this case, n = 1 and d = diameter of the telescope. Also, for small angle in radians, $\sin\theta = \theta$. So, we can now write:

$$\lambda = d\theta$$

.... *Equation 73*

And we can rearrange to give:

$$\theta = \frac{\lambda}{D}$$

.... *Equation 74*

Angle θ must be in **radians**. Make sure that your calculator is set to radians.

Worked Example

A camera lens is modelled as a single circular aperture of diameter 2.5 cm. It is used to photograph two light sources that both emit a wavelength of 520 nm. The sources are 1500 m away from the observer and can just be resolved.

(a) Work out the angle of resolution.
(b) How far apart are the sources?

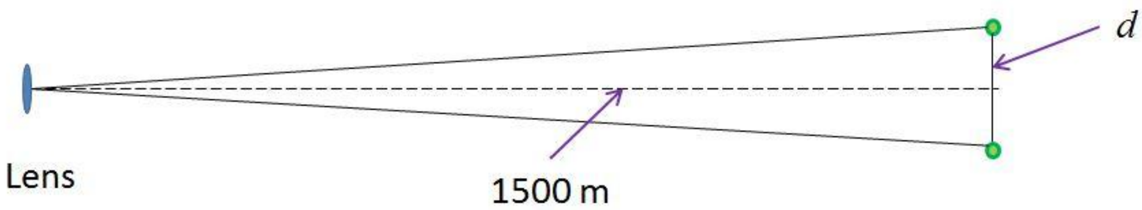
Answer

(a) It's a circular aperture, so we use:

$$\theta = 1.22\frac{\lambda}{D}$$

θ = 1.22 × (520 × 10^{-9} m ÷ 0.025 m) = 2.54 × 10^{-5} rad.

(b) Draw a diagram:



d = 1500 × $\sin\theta$ = 1500 m × 2.54 × 10^{-5} = 0.038 m ($\sin\theta \approx \theta$)

The separation is $2d$ = **0.076 m** (= 7.6 cm)

7.085 The Diffraction Grating

A **diffraction grating** can be used to split light into different wavelengths with a high degree of accuracy, much more so than glass prisms. A diffraction grating usually consists of a piece of glass with very closely spaced lines ruled on it. A **transmission** grating has clear spaces between the lines so that light can pass through it. A **reflection** grating has a shiny surface between the lines so that light gets reflected off it. A compact disc acts as a reflection grating.

The diffraction grating has the advantage over the double slit method of measuring wavelength in that:

- the maxima are more sharply defined.
- the beam passes through more slits than two, so the intensity is brighter.
- the angles are larger so that they can be measured with greater precision.

The derivation of the diffraction grating formula is on the syllabus, but experience has shown that most students struggle with it. It is more important that they know how to use the formula. If you want to see the derivation, it's in the next section. The formula is:

$$d \sin \theta = n\lambda \quad \text{............. Equation 75}$$

The term n is called the **spectrum order**. It is always a whole number. If n = 1, we have the **first diffraction** maximum. The other physics codes:

- θ is the angle,
- λ is the wavelength,
- and d is the slit width

Sin θ can never be greater than 1, so there is a limit to the number of spectra that can be obtained.

Worked Example A diffraction grating has 300 lines per mm. When it is illuminated normally by light of wavelength 530 nm, what is the angle between the first and second order maxima? What is the highest order maximum that can be obtained?
Answer Formula first: $n\lambda = d \sin \theta \Rightarrow \sin \theta = \frac{n\lambda}{d}$
There are 300 lines per mm, so there are 3×10^5 lines per metre. $\Rightarrow d = \frac{1}{3 \times 10^5 \text{ m}^{-1}} = 3.33 \times 10^{-6}$ m
Now put the numbers into the equation to work out the angle of the first order maximum: $\sin \theta = \frac{n\lambda}{d} = \frac{1 \times 530 \times 10^{-9} \text{ m}}{3.33 \times 10^{-6} \text{ m}} = 0.159 \Rightarrow \theta = \sin^{-1} (0.159) = 9.15^\circ$

Now put the numbers into the equation to work out the angle of the second order maximum:

$$\sin\theta = \frac{n\lambda}{d} = \frac{2\times 530\times10^{-9}\text{ m}}{3.33\times10^{-6}\text{ m}} = 0.318 \Rightarrow \theta = \sin^{-1}(0.318) = 18.54^{\circ}$$

So, the angle between the two maxima is 18.54 ° - 9.15 ° = **9.39°**

Now we can work out the highest order maximum by using sin *q* = 1:

$$1 = \frac{n\lambda}{d} \quad \Rightarrow n = \frac{d}{\lambda} = \frac{3.33\times10^{-6}\text{ m}}{530\times10^{-9}\text{ m}} = 6.3$$

Since the orders of maxima have to be **whole** numbers, the maximum order has to be **6**.

If the answer to the problem had been 6.87, the maximum order would still be 6, even though the nearest whole number was 7.

If we did further calculations to Question 7.08.2, we could see that the red light is diffracted more than blue light. The pattern would be like *Figure 105*.

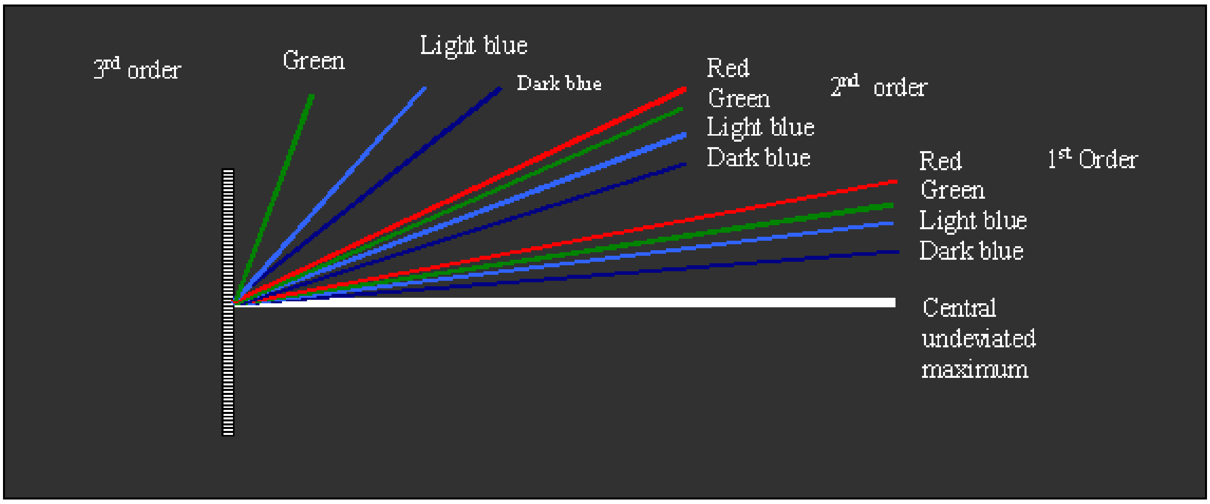


Figure 105 Red light is diffracted more than blue light

Note that:

- There is a central un-deviated maximum, which would be white light.
- The pattern would be symmetrical with orders either side of the maximum. We have not showed the ones below here.
- The third order does not have a red-light ray.

Derivation of the Diffraction Grating Formula

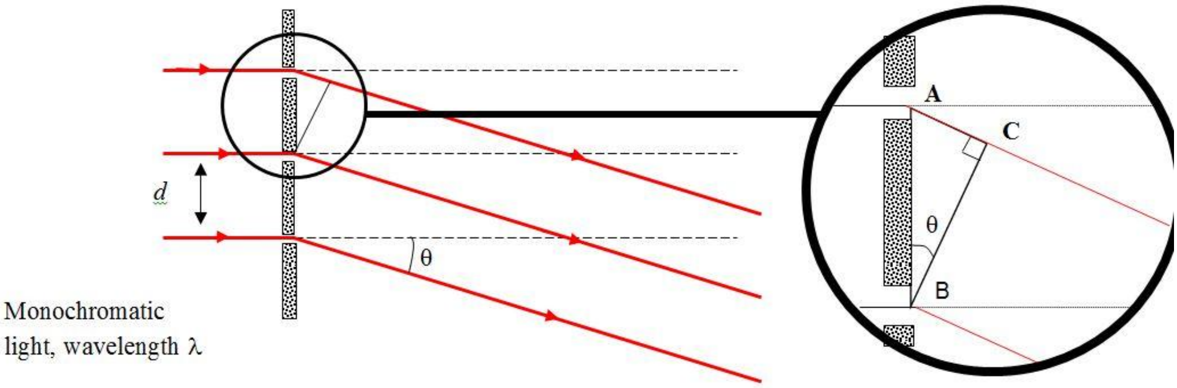


Figure 106 Monochromatic light passing through a diffraction grating

Parallel rays of a monochromatic light of wavelength λ are incident on a diffraction grating in which the slit separation is d. If the grating has N lines per metre, the grating spacing is given by:

$$d = \frac{1}{N}$$

.... *Equation 76*

Constructive interference only occurs along a few precise directions, one of which is shown in the diagram. Light from **A** must be in phase with light from **B**, and this can only happen when the path difference is a **whole number** of complete wavelengths (even number of half-wavelengths).

AC = $n\lambda$, *where* n = *0, 1, 2, 3…* ....*Equation 77*

Now:

$$\boldsymbol{AC} = d\ sin\ \theta \ldots\ldots\ldots. \quad \textit{Equation 78}$$

where θ is the angle of diffraction.

So, it does not take a genius to see that:

$$d \sin \theta = n\lambda \ldots\ldots\ldots. \quad \textit{Equation 79}$$

The term *n* is called the **spectrum order**. If *n* = 1, we have the **first diffraction** maximum.

Sin θ can never be greater than 1, so there is a limit to the number of spectra that can be obtained.

7.087 Diffraction Gratings and Spectra

The diffraction grating is a very good way of selecting light of a specific wavelength. Chemists and astronomers use diffraction gratings in **spectroscopy**, which allows them to see the specific **spectra** given out by different elements. Each element has its own individual spectrum. This allows astronomers to:

- See what elements there are in stars.
- If the spectrum "fingerprint" is shifted at all, astronomers can tell that a star is moving towards us (blue shift) or away from us (red shift). This is due to the **Doppler effect**.

The pictures (*Figure 107* and *108*) below show how spectra are used by astronomers. In *figure 107*, we can tell that, in the light emitted from the target star, there is water ice, methanol, methane gas, silicates and carbon dioxide. Each star's spectrum reveals its chemical composition. It's a stellar fingerprint.

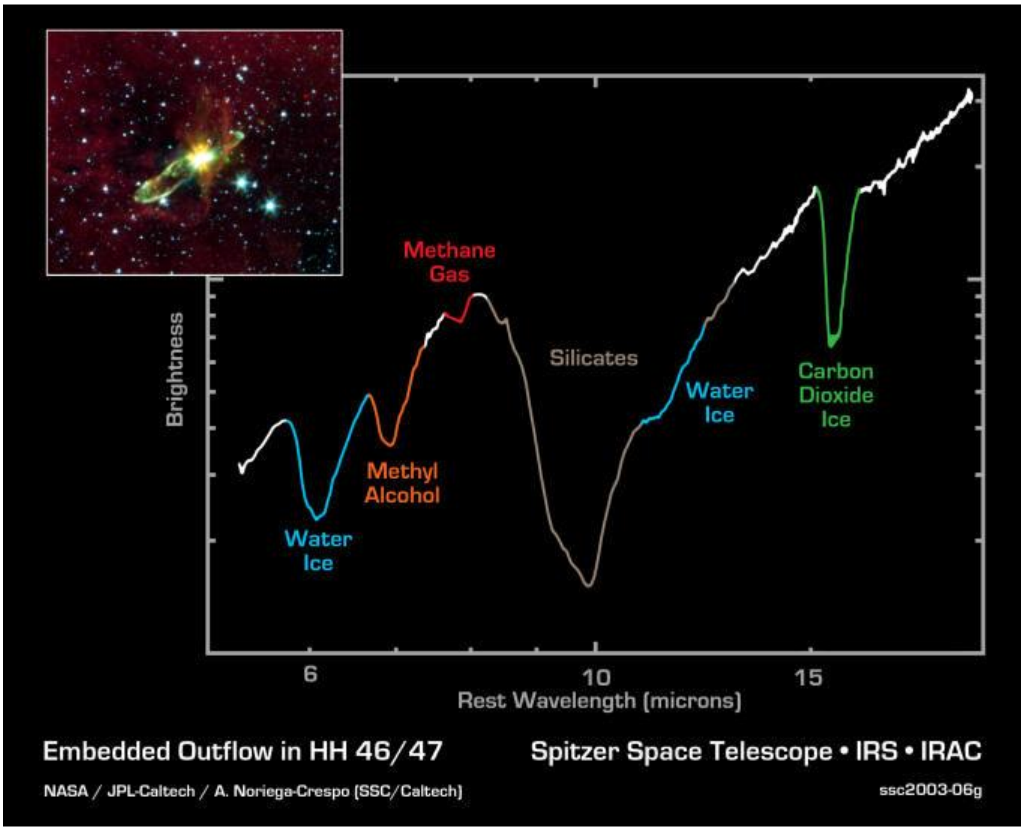


Figure 107 Spectroscopic analysis of light from stars (Photo courtesy of NASA, Wikimedia Commons.

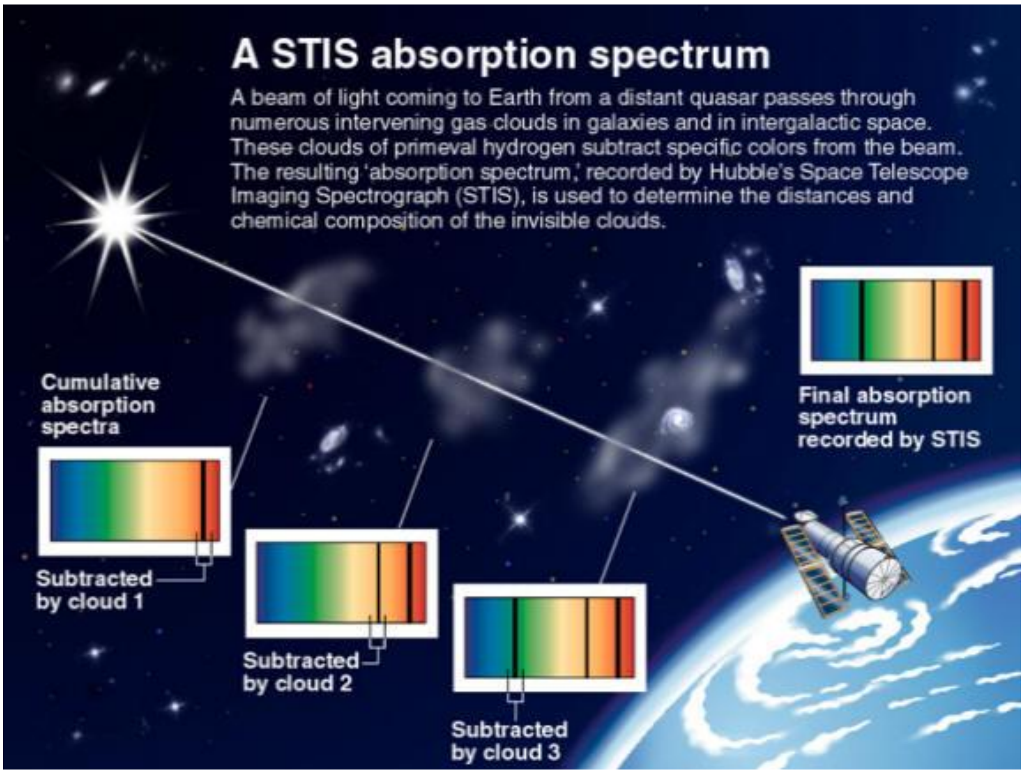


Figure 108 How subtraction of spectra can reveal chemical composition of invisible clouds (Picture courtesy of NASA, Wikimedia Commons

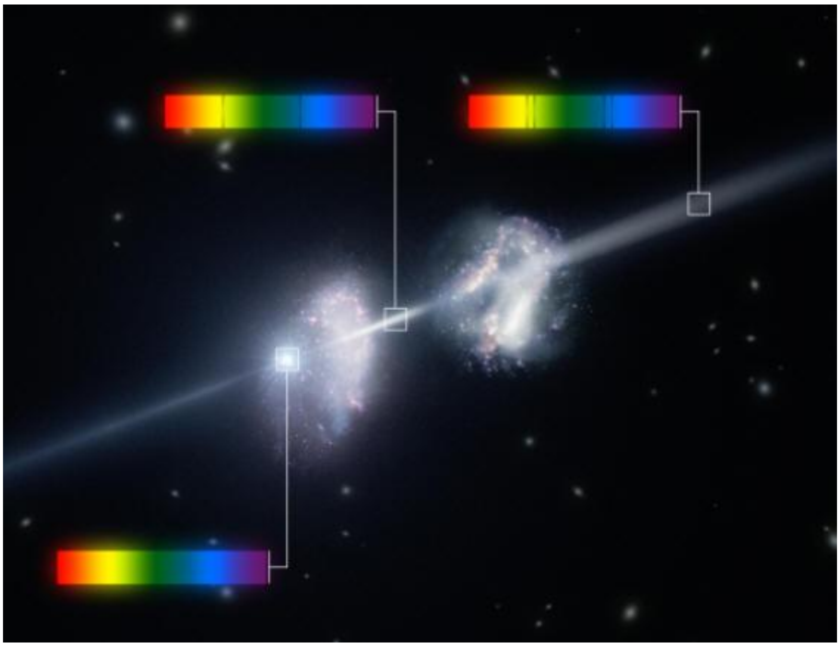


Figure 109 Spectroscopic images (Picture by ESO, Wikimedia Commons)

<u>Spectroscope</u>

Strictly speaking, there is a difference between **Spectroscopy** and **spectrometry**. **Spectroscopy** is the study of how a material radiates energy when it interacts with other matter. **Spectrometry** is the application of spectroscopy to obtain quantitative results in a particular spectrum. In this section, the section of the spectrum we are interested in this the **visible light spectrum**. At this level, I would doubt very much that you would lose marks for calling a spectrometer a "spectroscope".

In the school physics laboratory, the most common **spectroscope** is like this (*Figure 110*).

Figure 110 A school spectroscope

They are a simple tube with a **diffraction grating** at one end and an **eyepiece** at the other. Using these, we can observe emission lines from a glowing gas like hydrogen, or neon.

A more sophisticated instrument like the one shown below allows us to measure **angles of diffraction** (*Figure 111*).

Figure 111 A more sophisticated spectroscope.

This one uses a **glass prism**, although it can use a **diffraction grating**. The diagram below shows a spectroscope seen from above (*Figure 112*).

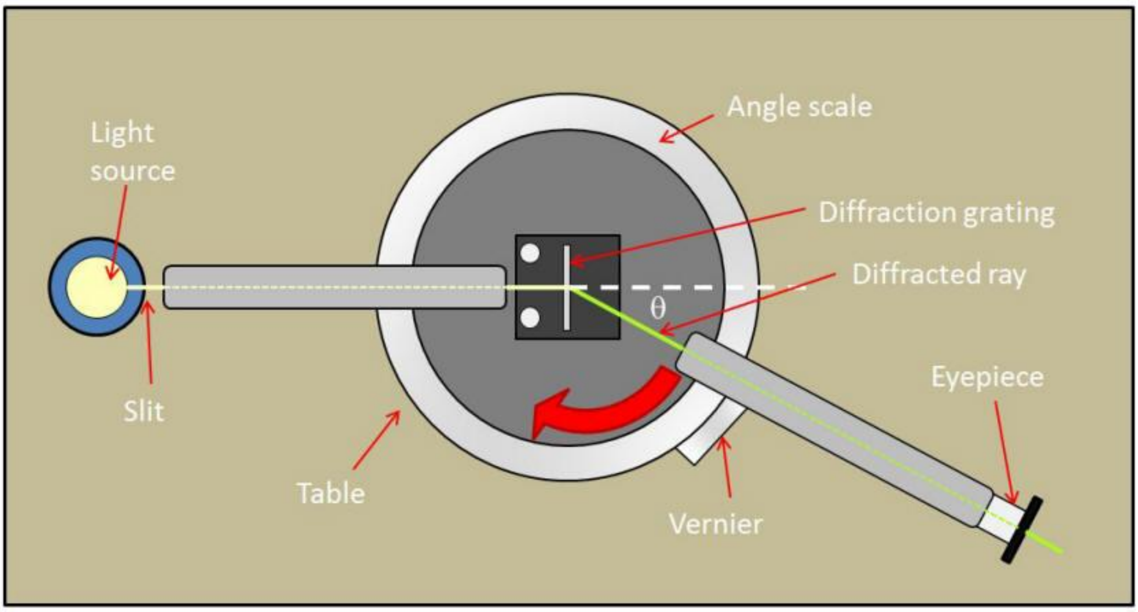


Figure 112 Spectroscope using a diffraction grating

Light from the source emerges through a **slit**. It passes through a tube to the **transmission diffraction grating**. The light is **diffracted** according to the wavelength. The eyepiece can travel around the table until the **first diffracted ray** is observed. The **angle** can be read off the scale. The **vernier** scale allows an accurate angle to be determined. Then the eyepiece is moved around the table until the next diffracted ray is seen. The data recorded in question 7.08.2 would have been obtained with a spectroscope like this. The pattern of diffracted rays would be this (*Figure 113*).

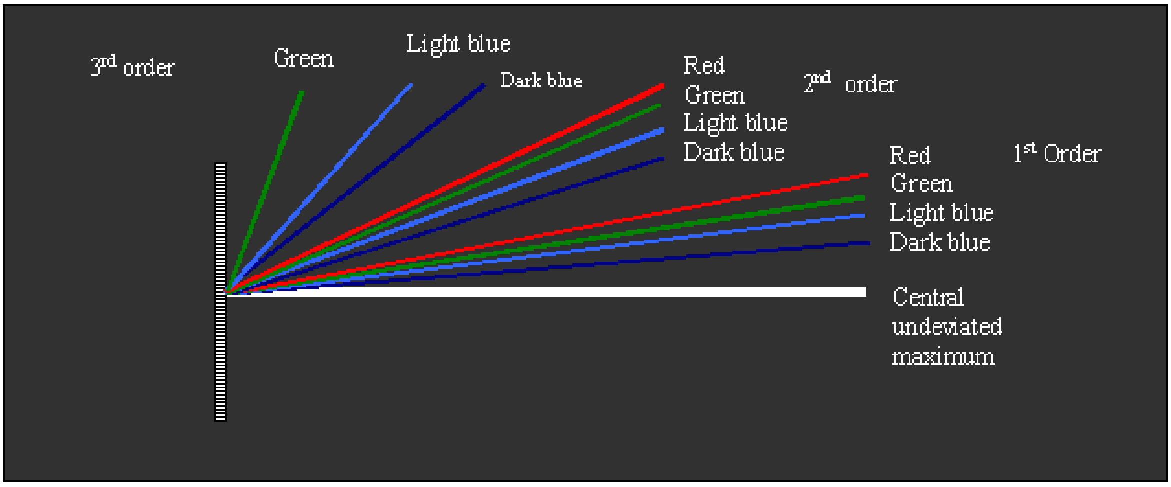


Figure 113 diffraction of cadmium light.

The **wavelength** of each diffracted ray would be determined by:

$$n\lambda = d \sin \theta \quad \text{………. } \textit{Equation 80}$$

The pattern in the diagram above is shown on one side only but is in reality **symmetrical**. Therefore, the eyepiece can move either side of the **central undeviated maximum**.

It is possible to have a spectroscope where the eyepiece, or **photodetector** is fixed and the diffraction grating is movable, like this (*Figure 114*).

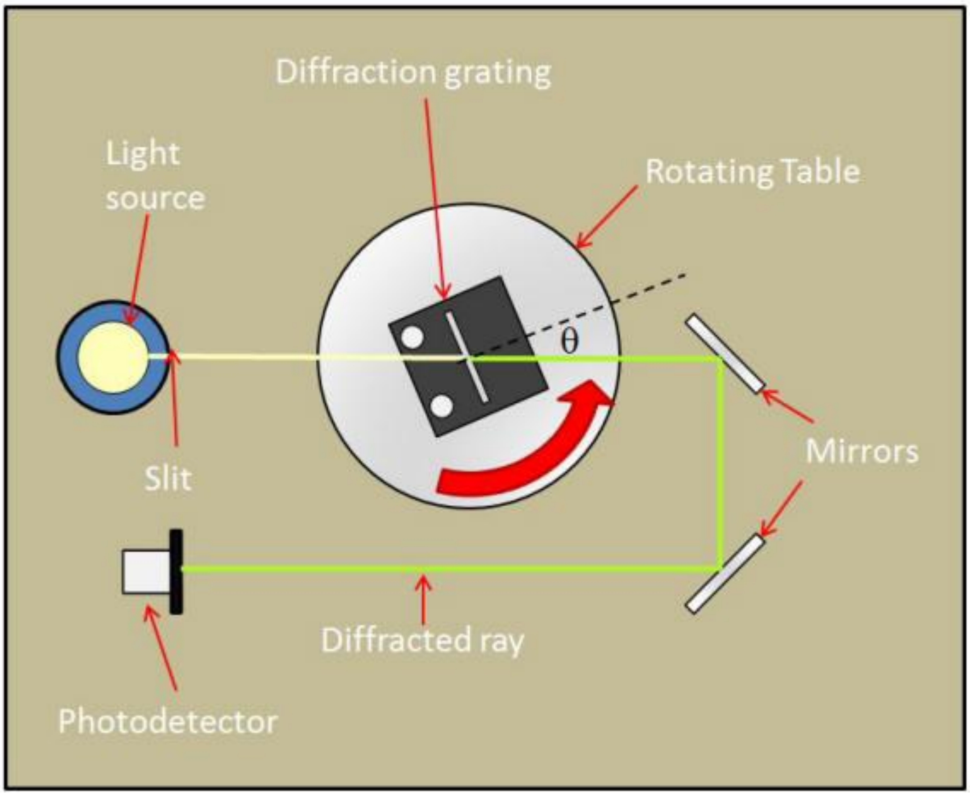


Figure 114 A spectrometer with a fixed phot detector.

In this case, the diffracted ray is selected by the diffraction grating and passes to the photodetector when the angle of the diffraction grating is correct. In some instruments, the knob that controls the rotating table is calibrated in **nanometres** (wavelength) rather than degrees.

The instrument in the diagram (*Figure 144*) uses a **transmission** grating, but many would use a **reflection** grating.

7.088 Resolvance (*IB students only*)

In a spectrum made by a **diffraction grating**, we may have two maxima that are very close together. Separation of the wavelengths is just possible when the maximum of one lies on the minimum of the next one, using **Rayleigh's Criterion**. The idea is shown below (*Figure 115*):

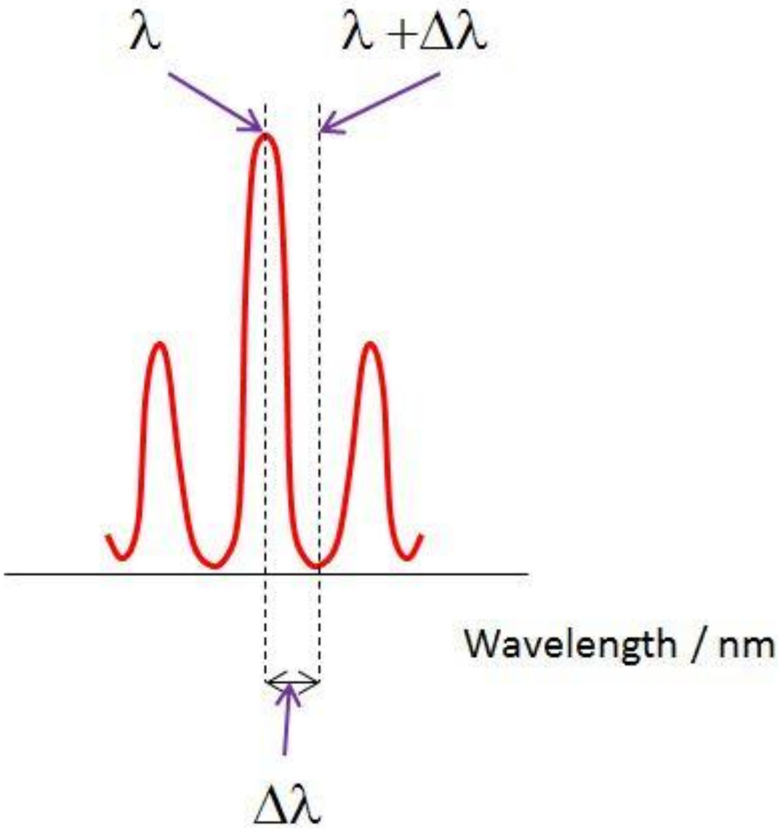


Figure 115 Resolvance of a diffraction grating

Resolvance is the **separating power** of a diffraction grating, or the **minimum separation between the wavelengths** that can be detected. The definition is:

Resolvance is the ratio of the wavelength to the minimum detectable difference of wavelength.

In Physics code, this is written:

$$R = \frac{\lambda}{\Delta\lambda}$$

.... *Equation 81*

The resolvance R has no units, since it is a ratio between wavelength (m) and difference in wavelength (m).

For a diffraction grating where there are N slits illuminated, the equation becomes:

$$R = \frac{\lambda}{\Delta\lambda} = mN$$

.... *Equation 82*

The term m is the order of the diffraction.

Worked example Two lines on a spectrum are known to have a wavelength of 525.05 nm and 525.87 nm. The first of these lines has a higher intensity than the second. (a) Calculate the resolvance. (b) Calculate the number of slits illuminated if this observation occurs at the first order of diffraction. (c) Calculate the slit width if the beam is 4.0 mm wide.
Answer (a) $\Delta\lambda$ = 525.87 nm - 525.05 nm = 0.82 nm R = 525.05 nm ÷ 0.82 nm = **640** (b) If m = 1, N = **640** (c) Slit width = 4.0 × 10^{-3} m ÷ 640 = **6.25 × 10^{-6} m**

Tutorial 7.08 Questions

7.08.1

The ASTRA satellite transmits radio waves (speed = 3×10^8 m/s) at a frequency of 12.5 GHz.

(1 GHz = 1×10^9 Hz)

(a) What is the wavelength of the signal?

(b) The transmitting dish is 1.6 m in diameter. Find the angle of diffraction.

(c) What is the radius of the circular patch that receives the signal? The height of the satellite is 3.6×10^7 m above the earth. (Ignore the curvature of the Earth)

7.08.2

When cadmium light is viewed through a diffraction grating having 500 lines per millimetre, the following spectral lines were observed at the stated angles.

Angle (degrees)	*Colour*
18.78	red
14.74	green
13.89	light blue
13.53	dark blue

Find the wavelength of these lines. Find the other angles at which spectral lines would be observed.

7.08.3

Why does the third order have no red ray?

7.08.4

A diffraction grating has 750 000 lines per metre.

(a) The beam of light passing through the diffraction grating is 2.0 mm wide. Calculate the resolvance for this grating for the first order of diffraction.

(b) A peak of red light is observed at 620 nm. Calculate the next wavelength upwards if the two wavelengths can be just resolved.

Tutorial 7.09 Colours	
Irish Syllabus	
Contents	
7.091 The EM Spectrum	7.092 Dispersion
7.093 Colour Addition	

This tutorial is for students of the Irish Syllabus

7.091 The Electromagnetic Spectrum

Visible light is a very small part of the **electromagnetic spectrum** (*Figure 116*).

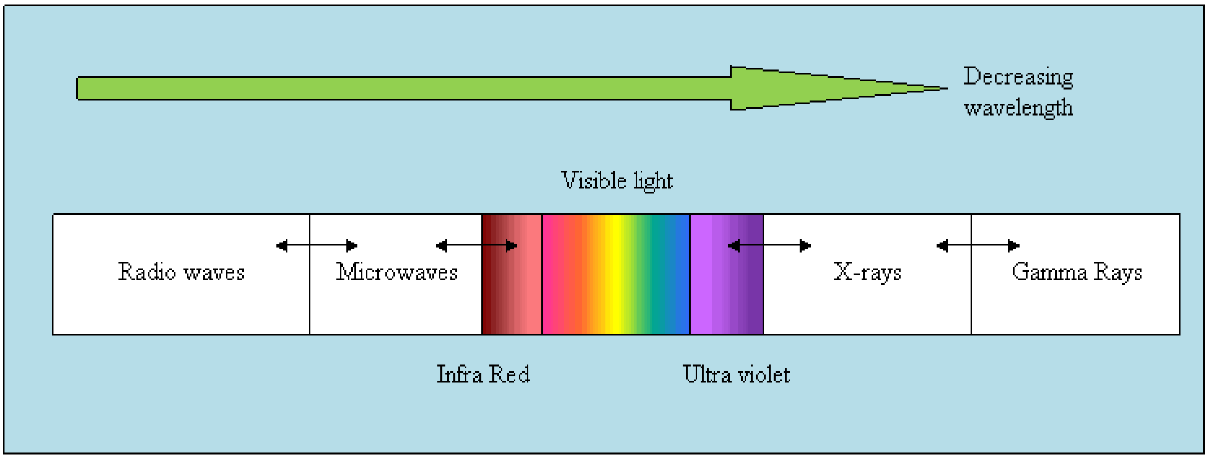


Figure 116 The EM spectrum

Insects can see wavelengths in the UV spectrum. A plain looking flower has elaborate markings when studied with a UV sensitive camera.

Many animals can see the whole spectrum from red light to blue light (700 nm to 400 nm), although some mammals can see just green and blue light. Tigers, being orange with black strips, may seem very obvious to us, but their prey can only see blue and green, so the tiger is very well camouflaged.

Most animals cannot see infra-red directly. Snakes have infra-red detectors, but these are not part of the eye. Biologists think that some fish, like goldfish, can see infra-red. Goldfish are the only animals that can see both UV and IR. Frogs are believed to be able to see IR.

7.092 Dispersion

If we pass monochromatic red light through a prism, we see that it is refracted like this (*Figure 117*)

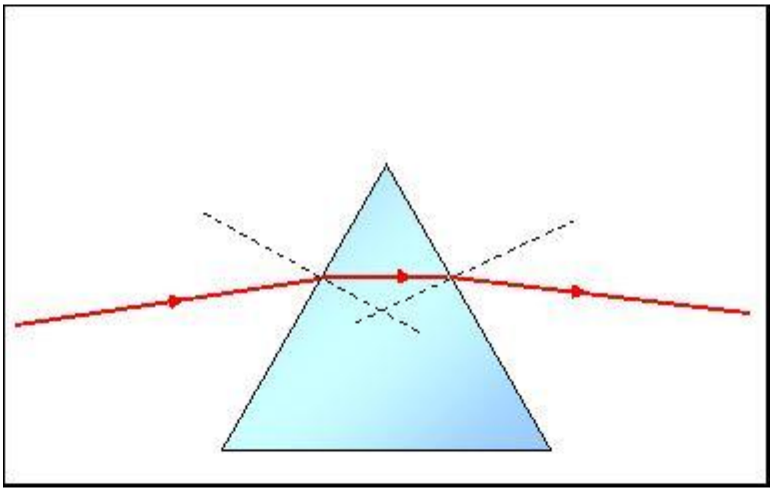


Figure 117 Refraction of monochromatic red light through a prism

If we use a ray of white light, we see that the light ray gets split into the colours of the rainbow (a spectrum). This is because different wavelengths **refract** by different amounts (*Figure 118*).

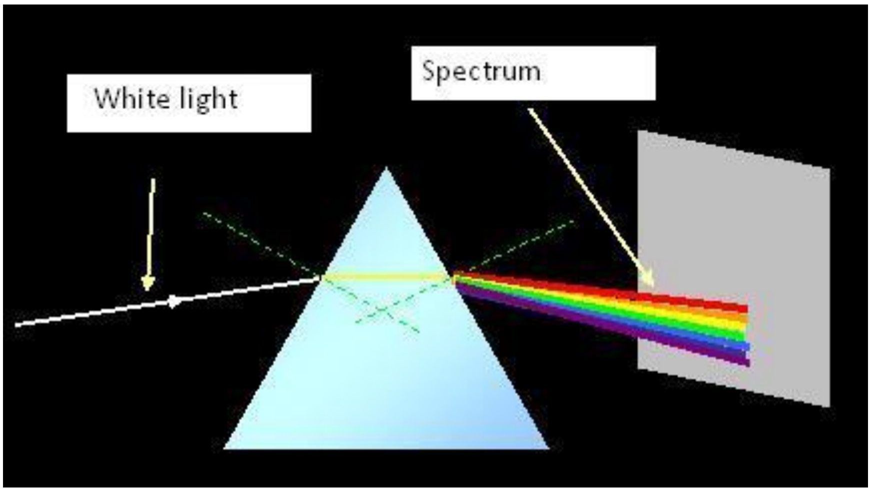


Figure 118 Dispersion of white light

This is called **dispersion**.

Dispersion explains **rainbows**. Droplets of water pick up white light from the sun. When sunlight strikes a raindrop, the ray will pass into the drop and the angle is greater than the **critical angle**, the ray will be reflected by **total internal reflection** on the water-air boundary (about 49°). The light will be reflected and will pass to the boundary, where it **refracts** as it emerges from the droplet. Remember that different colours refract by different amounts. So, we get a spectrum (*Figure 119*).

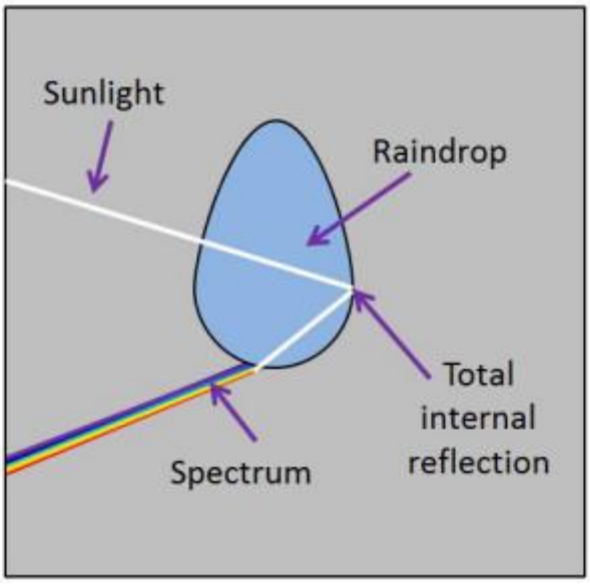


Figure 119 Dispersion in a raindrop

From that particular droplet, we may only see one colour, but there are millions of other droplets around the droplet we have considered. So, we see all the colours of the rainbow.

By using a second prism, we can combine all the rays to get a ray of white light (*Figure 120*).

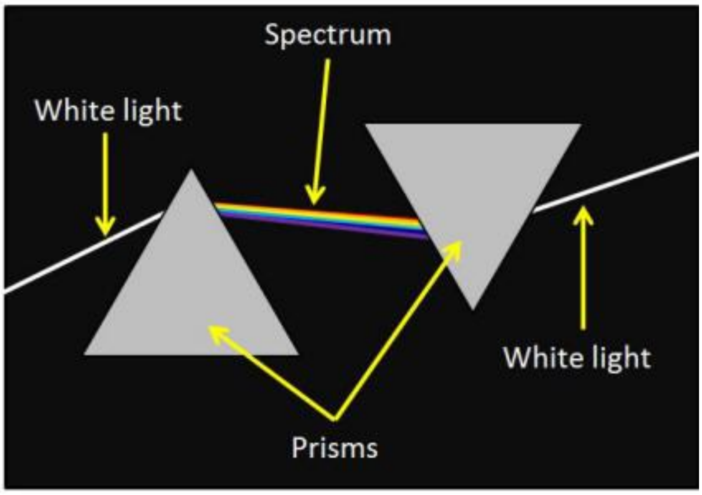


Figure 120 Recombining the spectrum back to white light

It is said that the young Isaac Newton showed dispersion by slitting one of the curtains in his mother's house and projecting the spectrum from the sunlight onto the wall. What his mother had to say about it is not recorded.

White light is all the colours of the rainbow added together.

7.093 Colour Addition

We can make white light from just three colours, **red**, **green**, and **blue**. These are called the **primary colours** (*Figure 121*).

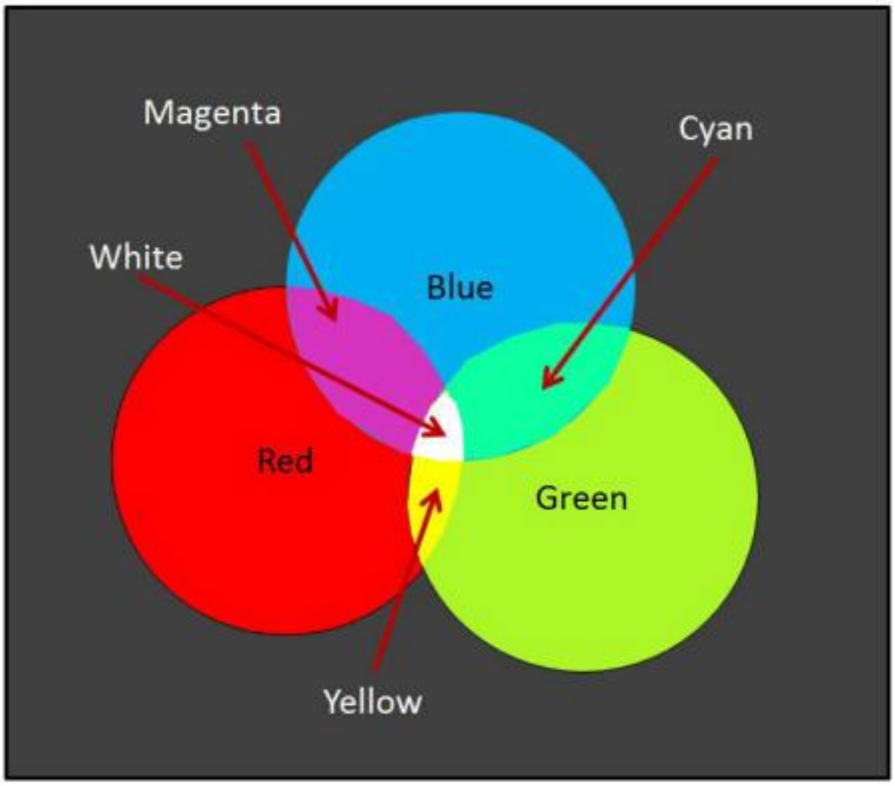


Figure 121 Combining red, green and blue light

Note that the primary colours are NOT the same as the artist's primary colours. The mixing of pigments involves colour **subtraction**, not colour addition. We will not consider colour subtraction here.

Secondary colours are:

- **cyan** - from green and blue.
- **magenta** - from red and blue.
- **yellow** - from red and green.

You can get a double light emitting diode that emits red when the first diode only is on, or green light when the second diode only is on. When both are on, the emitted light is yellow.

A **complementary** colour is one that you add to a **secondary** colour to give white light. The combinations are:

- yellow and blue.
- magenta and green.
- cyan and red.

The uses of colour addition include:

- stage lighting.
- colour television.

In television, the **photodetectors** in the camera are sensitive to red, green, and blue. A blue detector gives out its **maximum** response to blue light, and none to red or green. White light makes all three detectors give out their maximum response. Other colours, like yellow, will stimulate the red and green detectors, but not the blue. At the receiver, the individual pixels give out varying intensities of red, green, and blue, to make up the individual colours.

Tutorial 7.09 Questions

There are no questions for this tutorial.

Answers to Questions

Tutorial 7.01

7.01.1

Child on a swing

Bouncing mass on a spring

A vibrating speaker cone

7.01.2

Waves are caused by oscillations.

There is a maximum displacement...

...from the rest position.

At the maximum displacement, there is movement towards the rest position.

7.01.3

The particles move up and down (or forwards and backwards) ...

...which can do work.

The disturbance is travelling progressively.

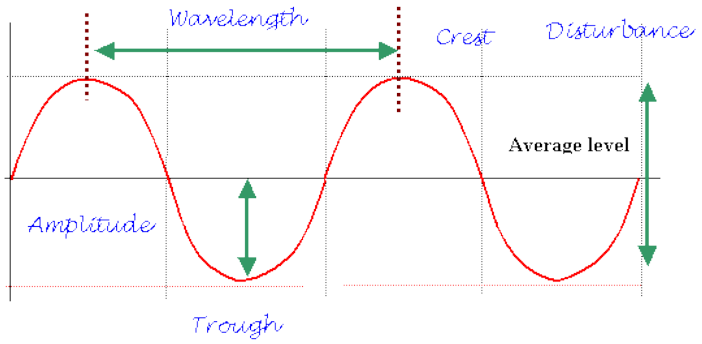
7.01.4

Displacement - Distance from the rest position (strictly speaking, above or below)

Period - Time taken for a complete cycle

Frequency - Number of cycles per second

7.01.5



7.01.6

(a)

Period of each wave = 2 s ÷ 8 = 0.25 s

f = 1 ÷ 0.25 s = **4 Hz**

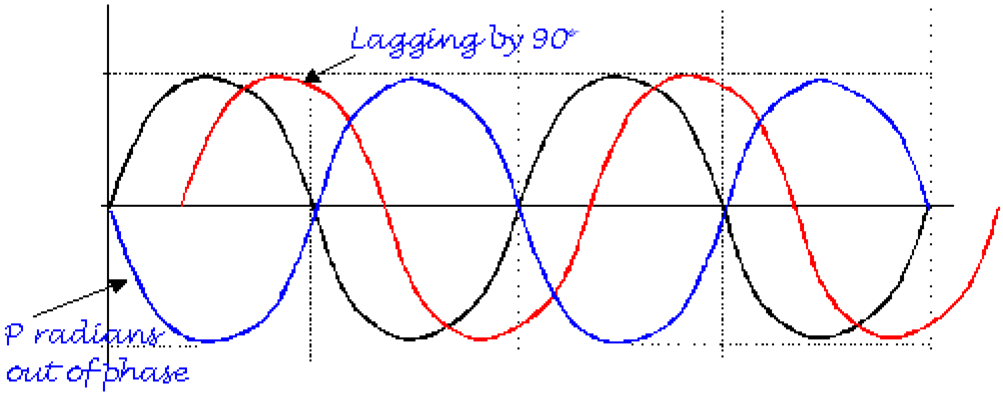
(b)

Wavelength = 48 cm ÷ 8 = **6 cm**

(c)

Speed $v = f\lambda$ = 4 Hz × 6 cm = **24 cm s^{-1}**.

7.01.7



Note: the label should say "Pi radians out of phase".

Tutorial 7.02

7.02.1

Similarities – Both carry energy from one point to another.

Both have a wavelength.

Difference – The movement in a transverse wave is at 90° to the motion while in the longitudinal, the displacement is parallel.

There are regions of compression and rarefaction in a longitudinal wave.

Example of a transverse wave: light waves.

Longitudinal wave: Sound wave.

7.02.2

a.

$$f = c/\lambda = 3.0 \times 10^{8}\text{ m s}^{-1} \div 95 \times 10^{-9}\text{ m} = \underline{\mathbf{3.2 \times 10^{15}\ Hz}}$$

b.

$$E = hc/\lambda = (6.63 \times 10^{-34}\text{ J s} \times 3.0 \times 10^{8}\text{ m s}^{-1}) \div 95 \times 10^{-9}\text{ m} = 2.093 \times 10^{-18}\text{ J}$$

$$= \underline{\mathbf{2.1 \times 10^{-18}\ J}}\text{ (2 s.f.)}$$

$$E = 2.093 \times 10^{-18}\text{ J} \div 1.6 \times 10^{-19}\text{ J eV}^{-1} = \underline{\mathbf{13.1\ eV}}$$

7.02.3

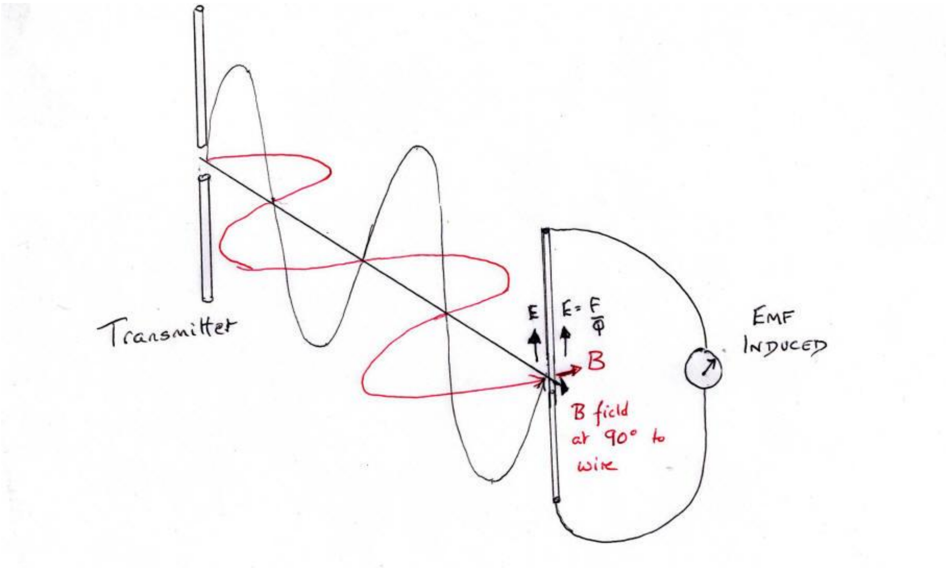
Radio waves are polarised...

...as they are transverse waves.

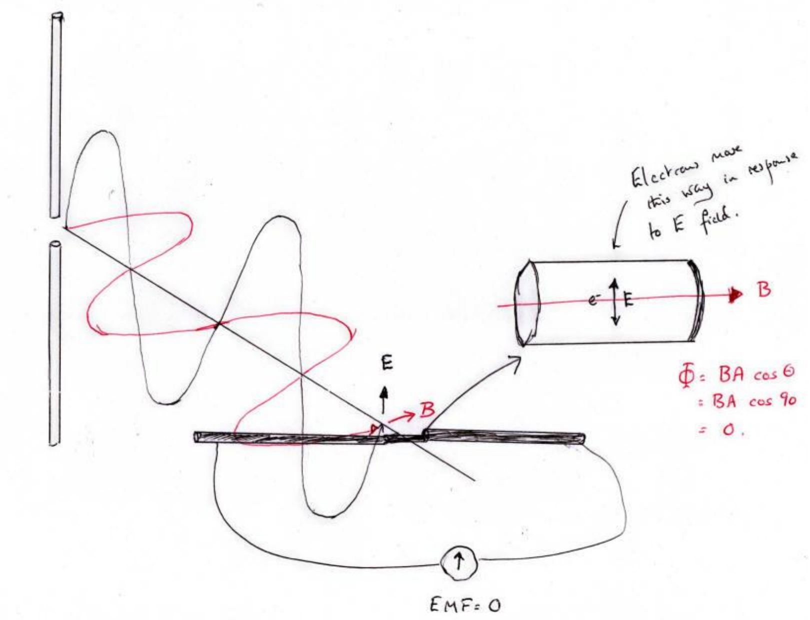
Polarisation only happens with transverse waves.

If the waves are vertically polarised the aerial needs to be vertical.

The diagrams below show the idea. The E field is vertical:



If the receiver aerial is at 90 degrees to the E-field, we see:



7.02.4

(a)

Period = 6 ms = 6×10^{-3} s

Frequency = **<u>167 Hz</u>**

(b)

Phase difference = π radians

because the crest of wave A meets a trough of wave B.

(c)

A longitudinal wave

because the speaker is compressing air molecules.

(d)

Loudspeaker A

because the amplitude is 2 mm (while B is 1 mm).

(e)

$c = f\lambda \Rightarrow \lambda = c/f$ = 340 m s^{-1} ÷ 167 Hz = **<u>2.04 m</u>**.

Tutorial 7.03

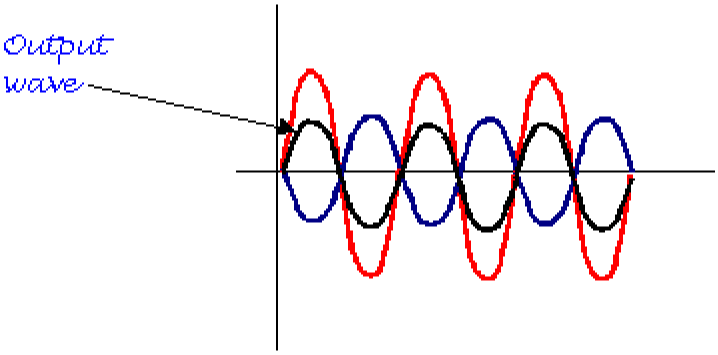
7.03.1

No

Although they are transverse, they are different wave types.

Light is electromagnetic; water is mechanical.

7.03.2



Tutorial 7.04

7.04.1

Equal frequency.

Nearly the same amplitude.

Same speed.

Travelling in opposite directions.

7.04.2

Node – a point of zero displacement in a standing wave;

Antinode - the point of maximum displacement.

Loop is ½ wavelength therefore the wavelength is 0.66 × 2 = **1.22 m**

7.04.3

Square the equation:

$$f^2 = \frac{1}{4l^2}\left(\frac{T}{\mu}\right)$$

Therefore:

$$T = 4\mu f^2 l^2$$

Now we need to find out the mass per unit length:

$\mu = 1.25 \times 10^{-3}$ kg ÷ 1.60 m = 7.8125×10^{-4} kg m^{-1}

(a) Now we can work out the tension:

$T = 4 \times 7.8125 \times 10^{-4}$ kg m^{-1} × (50.0 Hz)^2 × (1.15 m)^2 = **10.3 N**

(b)

v = (10.3 N ÷ 7.8125×10^{-4} kg m^{-1})$^{0.5}$ = **115 m s^{-1}**

7.04.4

Equation:

$$P = \frac{1}{2}\mu\omega^2 A^2 v$$

The angular velocity:

$\omega = 2\pi f = 2 \times \pi \times 50 \text{ Hz} = 314.15 \text{ rad s}^{-1}$

We know the mass per unit length:

$\mu = 1.25 \times 10^{-3} \text{ kg} \div 1.60 \text{ m} = 7.8125 \times 10^{-4} \text{ kg m}^{-1}$

(a) Now we can work out the power:

$P = 1/2 \times 7.8125 \times 10^{-4} \text{ kg m}^{-1} \times (2 \times \pi \times 50.0 \text{ Hz})^2 \times (20 \times 10^{-3} \text{ m})^2 \times 115 \text{ m s}^{-2}$

= 1.77 W = **1.8 W** (2 s.f.)

(b)

I = (1.77 W ÷ 2.5 V) = **0.71 A** (2 s.f.)

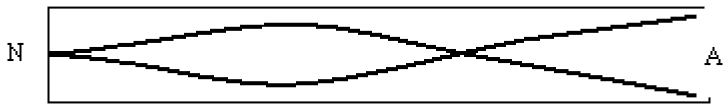
Tutorial 7.05

7.05.1

Any stringed instrument
The string is made to vibrate.
The note (frequency) is determined by how tight the string is;
how long it is;
and how thick it is.
The musician can alter the pitch by adjusting the tension.

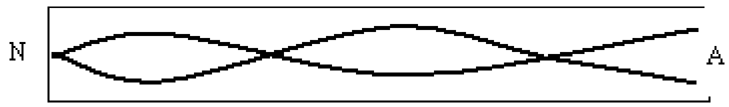
7.05.2

The next harmonic has will have a second node like this:



It is ¾ wavelength. Therefore, three times the frequency. This is the third harmonic.

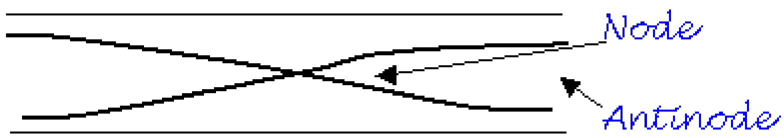
Then we get the next harmonic:



It is 1 ¼ wavelengths = 5/4 wavelength. It is five times the frequency, the fifth harmonic.

We get odd order harmonics.

7.05.3



We get second, third, fourth and fifth harmonics

7.05.4

(a)

It will be four octaves below concert pitch A

(b)

$\lambda = c/f$ = 340 m/s ÷ 22.5 Hz = **15.1 m**

(c)

The **closed pipe** arrangement will sound this note if the pipe is 3.8 m long.

This is because the fundamental standing wave is ¼ wavelength.

7.05.5

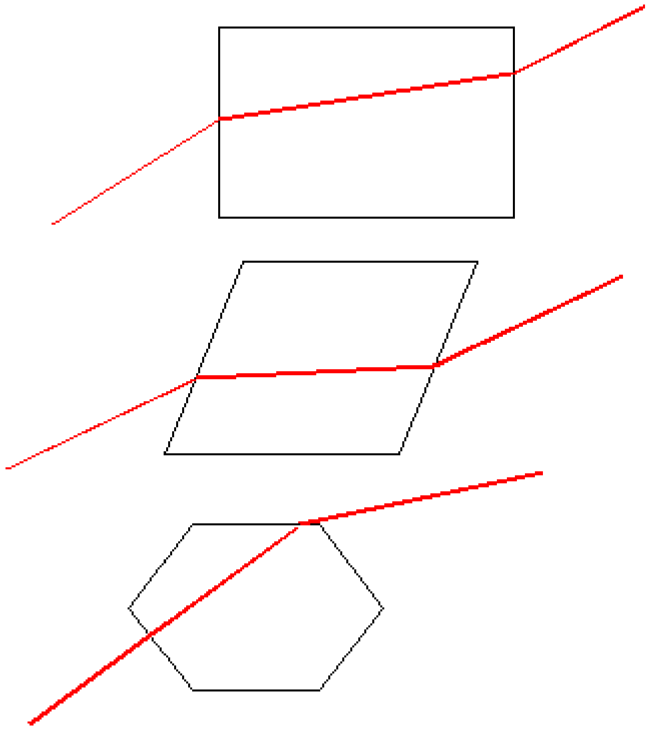
The standing wave is 1/4 of a wavelength

λ = 4 × 0.27 m = 1.08 m

$f = c/\lambda$ = 340 m s^{-1} ÷ 1.08 m = **315 Hz**

Tutorial 7.06

7.06.1



7.06.2

$n_2/n_1 = c_1/c_2$

1.5 ÷ 1.0 = 3.0 × 10^8 m s^{-1} ÷ c_2

c_2 = 3.0 × 10^8 m s^{-1} ÷ 1.5 = **2.0 × 10^8 m s^{-1}**

7.06.3

Going from material 2 to material 1.

7.06.4

${}_1n_2 = \sin\ \theta_1/\sin\theta_2$

1.50 = sin 30 ÷ sin θ_2 = 0.500 ÷ sin θ_2

sin θ_2 = 0.500 ÷ 1.5 = 0.333

θ_2 = $\sin^{-1}$ 0.333 = **19.5 °**

7.06.5

$$ {}_2n_1 = 1 \ / \ {}_1n_2 $$

$$ {}_{\text{glass}}\, n\, {}_{\text{air}} = 1 \div 1.5 = \mathbf{\underline{0.67}} $$

7.06.6

$$ \text{Use } {}_{\text{a}}n_{\text{w}} = \frac{\sin\ \theta_1}{\sin\ \theta_2} $$

[${}_{\text{a}}n_{\text{w}}$ is the refractive index passing from air to water.]

Put in the numbers:

$$ 1.33 = \frac{\sin 48^\circ}{\sin\ \theta_2} $$

$$ \sin\theta_2 = 0.743 \div 1.33 = 0.559 $$

$$ \theta_2 = \sin^{-1}(0.559) = 34^\circ $$

The angle of deviation is 48° – 34° = **<u>14°</u>**

7.06.7

$$ \text{Sin}\ \theta_c = 1/\ {}_1n_2 $$

$$ \sin\ \theta_c = 1/\ 1.33 = 0.75 $$

$$ \theta_c = \sin^{-1}\ 0.75 = \mathbf{\underline{48.6^\circ}} $$

7.06.8

Calculate the critical angle for a water and air boundary:

$$ \sin\ \theta_c = 1/1.33 = 0.75 $$

$$ \Rightarrow \theta_c = \sin^{-1}\ (0.75) = 48.8^\circ = \mathbf{\underline{49^\circ\ (\text{to 2 s.f.})}} $$

To work out r we need some trigonometry

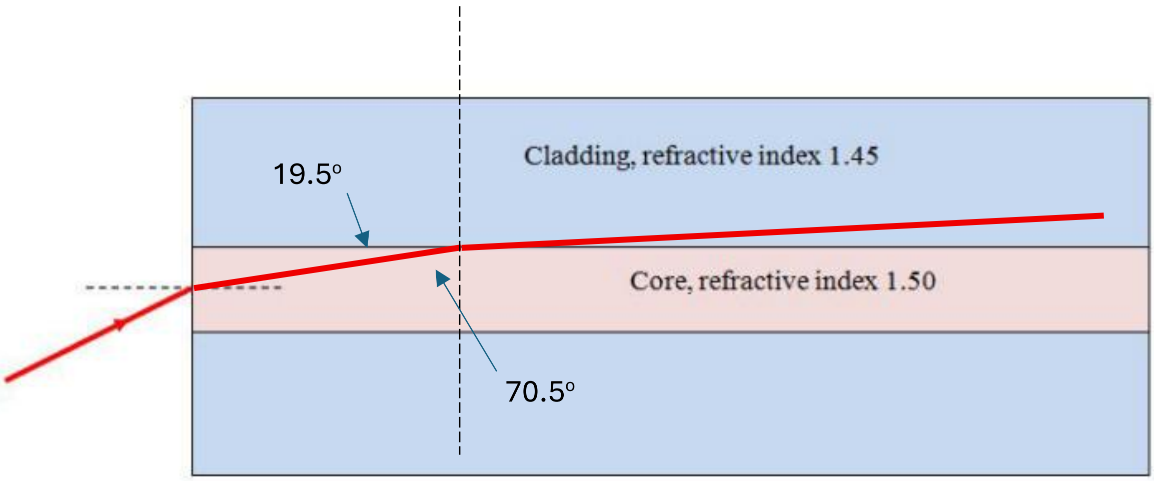
$$ r = 1.8\text{ m} \times \tan 48.8^\circ = 1.8 \times 1.14 = 2.05\text{ m} = \mathbf{\underline{2.1\text{ m (2 s.f.)}}} $$

7.06.9

(a) Use $n_1 \sin \theta_1 = n_2 \sin \theta_2$

$$1.00 \sin 30 = 1.50 \sin \theta_2$$

$$\theta_2 = \sin^{-1}(0.333) = \mathbf{\underline{19.5^\circ}}$$



(b) Critical angle:

$$1.50 \sin \theta_c = 1.45$$

$$\theta_c = \sin^{-1}(1.45 \div 1.50) = \sin^{-1}(0.967) = \mathbf{\underline{75^\circ}}$$

(c)

Angle of incidence at the boundary = 90° - 19.5° = 70.5°

This is less than the critical angle, therefore the ray will be refracted into the cladding.

Tutorial 7.07

7.07.1

It is the difference in the distance

between two wave sources and the points where the waves superpose.

It is measured in half-wavelengths.

7.07.2

For constructive interference we need an even number of half wavelengths.

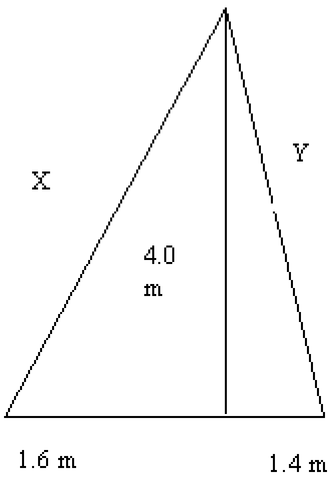
This means that two peaks or troughs will coincide.

For destructive interference we need odd numbers of half wavelengths.

So, a peak will coincide with a trough.

7.07.3

Move the microphone to the right by 10 cm. What is the path difference?



Path difference = $X - Y$

$X^2 = 4^2 + 1.6^2 = 16 + 2.56 = 18.56$

$X = 4.31$ m

$Y^2 = 4^2 + 1.4^2 = 16 + 1.96 = 17.96$

$Y = 4.24$ m

The path difference = 4.31 – 4.24 = 0.07 m

Since this is cancellation, the path difference is half a wavelength.

Therefore, the wavelength = 0.14 m

Frequency = speed ÷ wavelength = 340 m s^{-1} ÷ 0.14 m = **2400 Hz**

7.07.4

Sources of waves that are:

In phase.

Identical amplitudes.

Identical frequencies.

7.07.5

$\lambda = ws/D$

$w = D\lambda/s$ = (3.0 m × 630 × 10^{-9} m) ÷ 0.20 × 10^{-3} m

= **0.0095 m** (= 9.5 mm).

Tutorial 7.08

7.08.1

(a)

$$\lambda = c/f = 3 \times 10^8 \text{ m s}^{-1} \div 12.5 \times 10^9 \text{ Hz} = \underline{\mathbf{0.024\ m}} \text{ (= 2.4 cm)}$$

(b)

$$\sin\theta = \lambda/a = 0.024 \text{ m} \div 1.6 \text{ m} = 0.015$$

$$\theta = \sin^{-1} 0.015 = \underline{\mathbf{0.86^\circ}}$$

(c)

$$\text{Radius} = 3.6 \times 10^7 \text{ m} \times \tan 0.86 = 3.6 \times 10^7 \text{ m} \times 0.015$$

$$= \underline{\mathbf{540\,000\ m}} \text{ (= 540 km)}$$

7.08.2

Work out the spacing:

$$d = \frac{0.001 \text{ m}}{500} = 2.00 \times 10^{-6} \text{ m}^{-1}$$

Formula:

$$\lambda = \frac{d \sin\theta}{n}$$

For all of these angles, n = 1, so we can ignore it.

For red light:

$$\lambda = 2.00 \times 10^{-6} \times \sin 18.78^\circ = 6.44 \times 10^{-7} \text{ m} = \underline{\mathbf{644\ nm}}$$

We can do similar calculations to find the wavelengths of all the other spectral lines:

green - **<u>509 nm</u>**; light blue - **<u>480 nm</u>**; dark blue - **<u>468 nm</u>**.

To work out the second order angles we need to use:

$$\sin\theta = \frac{n\lambda}{d}$$

For red light:

$$\sin\theta = \frac{2 \times 6.44 \times 10^{-7} \text{ m}}{2.00 \times 10^{-6} \text{ m}} = 0.644$$

$$\Rightarrow \theta = \sin^{-1} 0.644 = \underline{\mathbf{40.09^\circ}} \text{ (not equal to } 2 \times 18.78^\circ)$$

7.08.3

If we used the equation $\sin\theta = n\lambda/d$,

we would find that $\sin\theta$ would be greater than 1.

This would be impossible, so there is no diffraction possible.

7.08.4

(a) Work out the slit width:

d = 1 ÷ 750 000 m^{-1} = 1.33 × 10^{-6} m.

in 2.0 × 10^{-3} m,

N = 2.0 × 10^{-3} m ÷ 1.33 × 10^{-6} m = 1500

$R = Nm$ = 1500 × 1 = **1500**

(b) Use:

$$R = \frac{\lambda}{\Delta\lambda}$$

$\Delta\lambda$ = 620 × 10^{-9} m ÷ 1500 = 0.413 × 10^{-9} m.

The next wavelength upwards that can just be resolved

= 620 × 10^{-9} m + 0.413 × 10^{-9} m = 620.413 × 10^{-9} m = **620.413 nm**